1. Use the Binomial Theorem to expand $(x^3-2)^5$.

$$\binom{5}{0}(x^3)^5+\binom{5}{1}(x^3)^4(-2)^1+\binom{5}{2}(x^3)^3(-2)^2+\binom{5}{3}(x^3)^2(-2)^3+\binom{5}{4}(x^3)^1(-2)^4+\binom{5}{5}(-2)^5$$

$$=1\cdot x^{15}+5\cdot x^{12}\cdot(-2)+10\cdot x^9\cdot 4+10\cdot x^6\cdot(-8)+5\cdot x^3\cdot 16+1\cdot(-32)$$

$$=\boxed{x^{15}-10x^{12}+40x^9-80x^6+80x^3-32}$$

$$\binom{5}{2}=\frac{5!}{2!\,3!}=\frac{5\cdot 4}{2\cdot 1}=10$$

$$\binom{5}{3}=\frac{5!}{3!\,2!}=10$$

2. Find the coefficient of x^4 in the expansion of $(3x-1)^6$. $\leftarrow n=6$

$a=3x \quad b=-1$

$$\binom{n}{k}a^{n-k}b^k$$

$$\binom{6}{k}(3x)^{6-k}(-1)^k$$

$x^{6-k}=x^4$

$6-k=4$

$k=2$

$$\binom{6}{2}(3x)^{6-2}(-1)^2$$

$$(15)(3x)^4(1)$$

$$1215x^4$$

$$\binom{6}{2}=\frac{6!}{2!\,4!}=\frac{6\cdot 5}{2\cdot 1}=15$$

Coefficient of x^4 term: $\boxed{1215}$

Q: What are the next two letters in this sequence: A E F H I K L M ?

3. Use the Binomial Theorem to expand $(2x-3)^6$.

$$\binom{6}{0}(2x)^6+\binom{6}{1}(2x)^5(-3)^1+\binom{6}{2}(2x)^4(-3)^2+\binom{6}{3}(2x)^3(-3)^3+\binom{6}{4}(2x)^2(-3)^4+\binom{6}{5}(2x)^1(-3)^5+\binom{6}{6}(-3)^6$$

$$=1\cdot 64x^6+6\cdot 32x^5\cdot(-3)+15\cdot 16x^4\cdot 9+20\cdot 8x^3\cdot(-27)+15\cdot 4x^2\cdot 81+6\cdot 2x\cdot(-243)+1\cdot 729$$

$$=\boxed{64x^6-576x^5+2160x^4-4320x^3+4860x^2-2916x+729}$$

5. Use the Binomial Theorem to find the first three terms in the expansion of $\left(\frac{1}{x}+1\right)^{17}$.

$$\binom{17}{0}\left(\frac{1}{x}\right)^{17}+\binom{17}{1}\left(\frac{1}{x}\right)^{16}(1)^1+\binom{17}{2}\left(\frac{1}{x}\right)^{15}(1)^2+\dots$$

$$=1\cdot\frac{1}{x^{17}}+17\cdot\frac{1}{x^{16}}\cdot 1+136\cdot\frac{1}{x^{15}}\cdot 1+\dots$$

$$=\boxed{\frac{1}{x^{17}}+\frac{17}{x^{16}}+\frac{136}{x^{15}}}+\dots$$

7. Find the coefficient of x^5 in the expansion of $(x-2)^9$.

$$\binom{n}{k}a^{n-k}b^k$$

$$\binom{9}{k}x^{9-k}(-2)^k$$

$x^{9-k}=x^5$

$9-k=5$

$k=4$

$$\binom{9}{4}x^{9-4}(-2)^4$$

$$\binom{9}{4}=\frac{9!}{4!\,5!}=\frac{\overset{3}{\cancel{9}}\cdot\cancel{8}\cdot 7\cdot 6}{\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot 1}=126$$

$$(126)x^5(16)$$

$$2016x^5$$

Coefficient: $\boxed{2016}$