Sequences

(covers Sullivan 12.1)

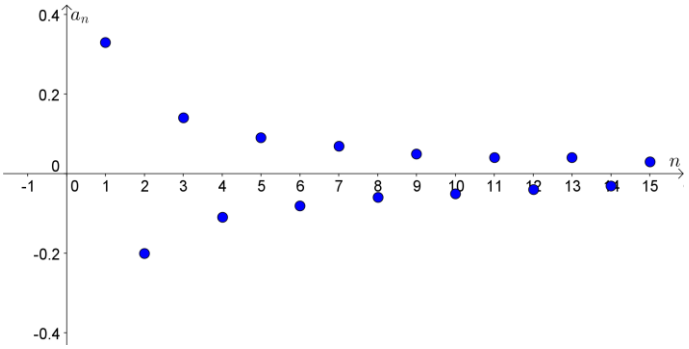
A sequence is an infinite list of numbers $a_1, a_2, a_3, \ldots, a_n, \ldots$
ex: $3, \quad 7, \quad 11, \quad 15, \quad 19, \ldots$

ex: $a_n = \frac{n}{n+1}$ is a compact way of writing the sequence $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$

Ex 1.

Find the first three terms of $a_n = \frac{(-1)^{n+1}}{2n+1}$. Then find the 100th term.

We can visualize sequences as a bunch of dots on a coordinate system, with n on the x-axis, and a_n on the y-axis. Here's the sequence from Ex 1:



A **recursive sequence** is one where each term depends on one or more terms before it.

Ex 2.

Find the first four terms of the sequence defined recursively by $a_1 = 1, \quad a_n = a_{n-1} + 2$.

Ex 3.
Find the first three terms of the sequence defined recursively by $a_1 = -1, \quad a_n = \frac{a_{n-1}+6}{a_{n-1}+2}$

In Math 181, it will be useful to add the terms of a sequence up.
The **partial sums** of a sequence $a_1, a_2, a_3, \ldots, a_n, \ldots$ are defined as:

$S_1 = a_1$
$S_2 = a_1 + a_2$
$S_3 = a_1 + a_2 + a_3$
$S_4 = a_1 + a_2 + a_3 + a_4$
…
$S_n = a_1 + a_2 + a_3 + \cdots + a_n$

Ex 4.
Find the first four partial sums of the sequence $a_n = 1/2^n$.

Summation notation (also called **sigma notation**) is a compact way of writing partial sums.

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + a_4 + \cdots + a_n$$

Ex 5.
Find the sum.

$$\sum_{k=1}^{4} (k^2 - 3k)$$

Ex 6.
Find a formula for the nth term of the sequence $1, -\frac{1}{4}, \frac{1}{9}, -\frac{1}{16}, \frac{1}{25}, \ldots$

Let's look at some common patterns in sequences:

n:	$1, 2, 3, 4, 5, 6, \ldots$	(Note: can add 1 to get next term)
$2n$:	$2, 4, 6, 8, 10, 12, \ldots$	(Note: can add 2 to get next term)
$3n$:	$3, 6, 9, 12, 15, \ldots$	(Note: can add 3 to get next term)
$5n$:	$5, 10, 15, 20, 25, \ldots$	(Note: can add 5 to get next term)
$5n + 1$:	$6, 11, 16, 21, 26, \ldots$	(Note: can still add 5 to get next term)
$5n - 3$:	$2, 7, 12, 17, 22, \ldots$	(Note: can still add 5 to get next term)
$-2n$:	$-2, -4, -6, -8, -10, \ldots$	(Note: can subtract 2 to get next term)
n^2:	$1, 4, 9, 16, 25, 36, \ldots$	
n^3:	$1, 8, 27, 64, 125, \ldots$	
2^n:	$2, 4, 8, 16, 32, 64, \ldots$	(Note: can multiply by 2 to get next term)
3^n:	$3, 9, 27, 81, 243, \ldots$	(Note: can multiply by 3 to get next term)
3^{n+1}:	$9, 27, 81, 243, 729, \ldots$	(Note: can still multiply by 3 to get next term)
3^{n-1}:	$1, 3, 9, 27, 81, 243, \ldots$	(Note: can still multiply by 3 to get next term)
$(-1)^n$:	$-1, 1, -1, 1, -1, 1, \ldots$	
$(-1)^{n+1}$:	$1, -1, 1, -1, 1, -1, \ldots$	

Ex 7.
Find a formula for the nth term of the sequence $-\frac{1}{5}, \frac{2}{8}, -\frac{4}{11}, \frac{8}{14}, -\frac{16}{17}, \frac{32}{20}, \ldots$

Ex 8.
Write the sum using sigma notation.
$1 - \frac{7}{9} + \frac{11}{27} - \frac{15}{81} + \frac{19}{243}$

Ex 9.
Write the sum using sigma notation.
$\sqrt{4} + \sqrt{5} + \sqrt{6} + \cdots + \sqrt{88}$