

Partial Fraction Decomposition

(covers Sullivan 11.5)

Let's look at a technique to break rational functions up into the sum of simpler rational functions.

For example: $\frac{5x-3}{x^2-2x-3} = \frac{5x-3}{(x+1)(x-3)} = \frac{2}{x+1} + \frac{3}{x-3}$

$\frac{2}{x+1} + \frac{3}{x-3}$ is called the **partial fraction decomposition** of $\frac{5x-3}{x^2-2x-3}$.

Ex 1.

Find the partial fraction decomposition of:

$$\frac{x^2+4x+1}{(x-1)(x+1)(x+3)}$$

In general, we need to factor the denominator first. What happens if we don't get linear factors?

1. If we get a **repeated linear factor** $(ax + b)^k$, then we'll have corresponding partial fractions:

$$\frac{A_1}{ax+b} + \frac{A_2}{(ax+b)^2} + \cdots + \frac{A_k}{(ax+b)^k}$$

2. If we get an **irreducible quadratic factor** $ax^2 + bx + c$, then we'll have a corresponding partial fraction: $\frac{Ax+B}{ax^2+bx+c}$

3. If we get a **repeated irreducible quadratic factor** $(ax^2 + bx + c)^k$, then we'll have corresponding partial fractions: $\frac{A_1x+B_1}{(ax^2+bx+c)} + \frac{A_2x+B_2}{(ax^2+bx+c)^2} + \cdots + \frac{A_kx+B_k}{(ax^2+bx+c)^k}$

Ex 2.

Write the form of the partial fraction decomposition of $\frac{x^5-3x^2+12x-1}{x^3(x^2+x+1)(x^2+2)^3}$.

Ex 3.

Find the partial fraction decomposition of $\frac{6x+7}{(x+2)^2}$.

Ex 4.

Find the partial fraction decomposition of $\frac{-2x+4}{(x^2+1)(x-1)^2}$.

Note: If degree of top polynomial is ___ degree of bottom polynomial, then _____ first.

Ex 5.

Find the partial fraction decomposition of $\frac{2x^3-4x^2-x-3}{x^2-2x-3}$.