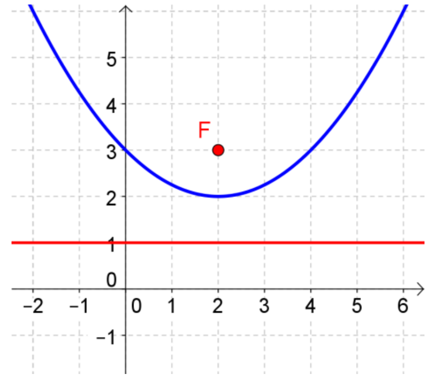
Conic Sections (Parabolas, Ellipses, Hyperbolas)

(covers parts of Sullivan 10.1, 10.2, 10.3, and 10.4)



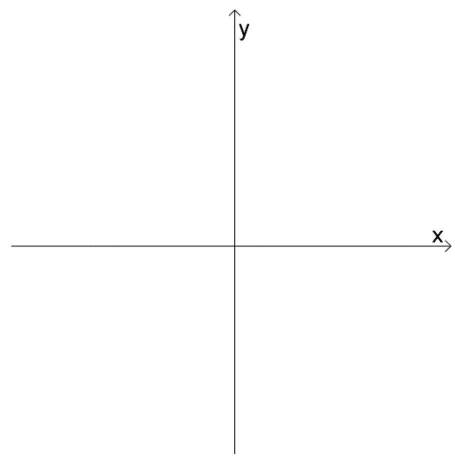
A **parabola** is defined as the set of all points in a plane equidistant from a given fixed point (called the **focus**) and a given fixed line (called the **directrix**).

$x^2 = 4py$ has focus $(0, p)$ and directrix $y = -p$.
(if $p > 0$ opens up, if $p < 0$ opens down)

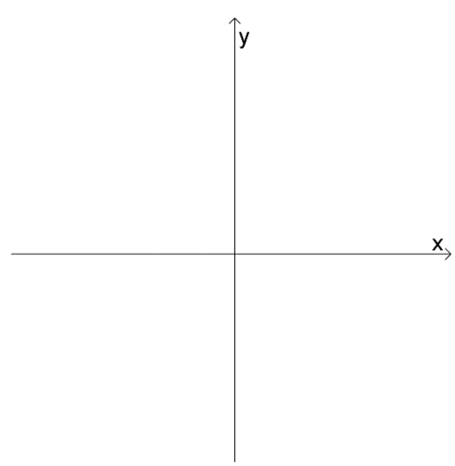
$y^2 = 4px$ has focus $(p, 0)$ and directrix $x = -p$.
(if $p > 0$ opens right, if $p < 0$ opens left)

The **focal diameter** is the distance across the parabola through the focus. It is: $|4p|$

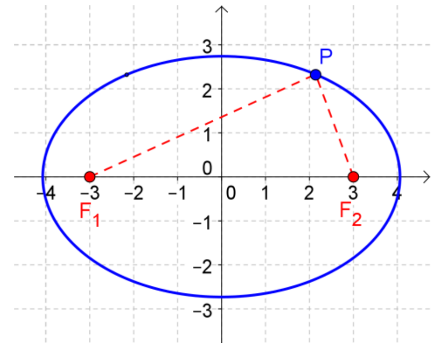
Ex 1.
Find the focus, directrix, and focal diameter of the parabola, and sketch its graph.
$y^2 = -2x$



Ex 2.
Find the focus, directrix, and focal diameter of the parabola, and sketch its graph.
$x^2 + y = 0$



An **ellipse** is defined as the set of all points in a plane whose distances from two fixed points (called **foci**) in the plane have a constant *sum*.



The equation for an ellipse centered at the origin is:

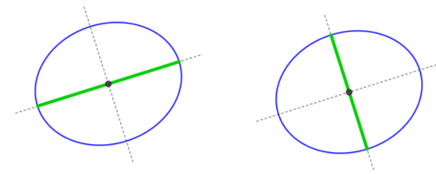
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

The x-intercepts are $x=\pm a$, and the y-intercepts are $y=\pm b$.

The foci have a distance of $c=\sqrt{a^2-b^2}$ from the center.

(or if $b>a$, then $c=\sqrt{b^2-a^2}$)

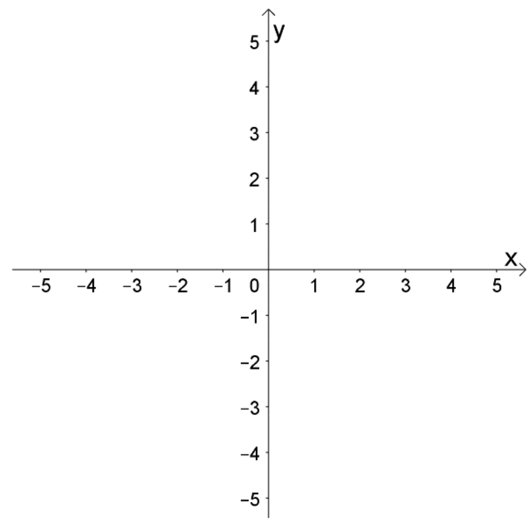
The **vertices** are the points at the ends of the major axis.



Ex 3.

Find the vertices and foci of the ellipse, determine the lengths of the major and minor axes, and sketch its graph.

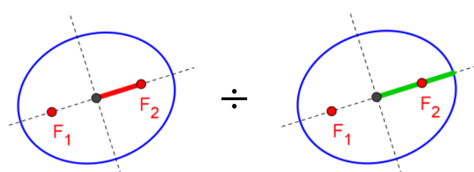
$25x^2+16y^2=400$



The **eccentricity** of an ellipse measures how much the ellipse deviates from a circle. It's the ratio of the focal length and the longer semiaxis.

If $a>b$, then $e=\frac{c}{a}$. If $b>a$, then $e=\frac{c}{b}$.

Eccentricity =



(Note: For all ellipses, $0<e<1$. If more circular, e is closer to 0. If more elongated, e is closer to 1.)

Ex 4.

Find the eccentricity of the ellipse: $\frac{x^2}{9}+\frac{y^2}{4}=1$

A **hyperbola** is the set of points in a plane whose distances from two fixed points (called **foci**) in the plane have a constant *difference*.

The equation for a hyperbola centered at the origin opening *horizontally* is:

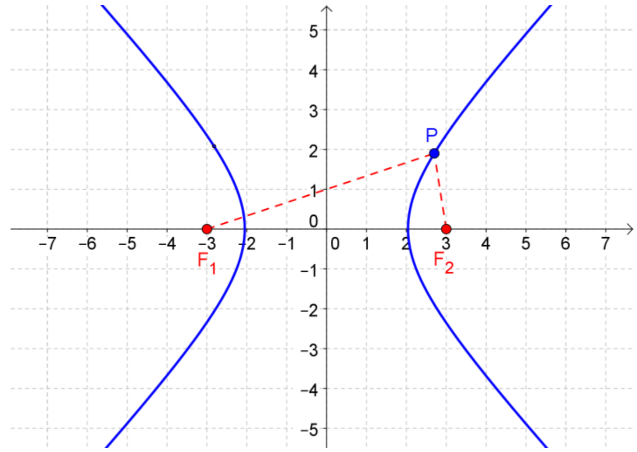
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

The x-intercepts are $x = \pm a$.

The foci have a distance of $c = \sqrt{a^2 + b^2}$ from the center.

The hyperbola has asymptotes $y = \pm \frac{b}{a} x$.

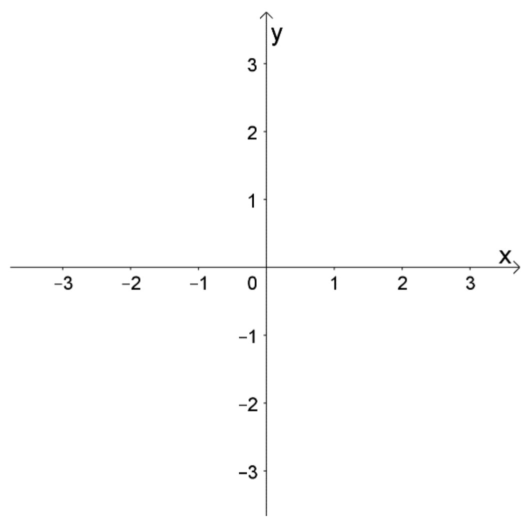
The **vertices** are $(\pm a, 0)$.



Ex 5.

Find the vertices, foci, and asymptotes of the hyperbola, and sketch its graph.

$x^2 - 3y^2 = 3$



The equation for a hyperbola centered at the origin opening *vertically* is:

$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$

The y-intercepts are $y = \pm b$.

The foci have a distance of $c = \sqrt{a^2 + b^2}$ from the center.

The hyperbola has asymptotes $y = \pm \frac{b}{a} x$.

The **vertices** are $(0, \pm b)$.

Note: All conics can be shifted to have center (h, k) by replacing x with $x - h$, and y with $y - k$.

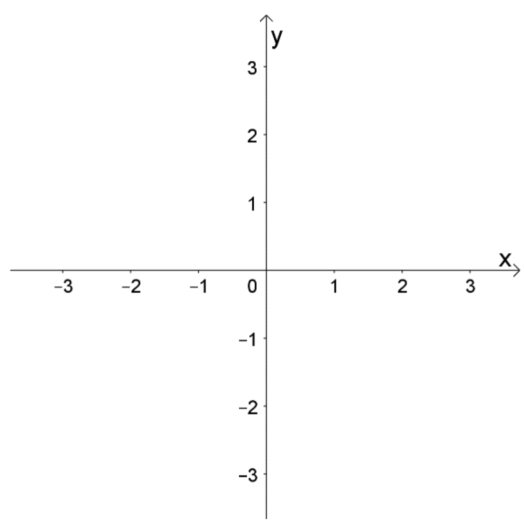
ex: $\frac{(x-2)^2}{4} + \frac{(y+1)^2}{9} = 1$ is an ellipse with center $(2, -1)$.

Ex 6.

Determine whether the equation represents a parabola, an ellipse, or a hyperbola.

$y^2 - 4x^2 + 2y + 8x - 7 = 0$

Find the center, foci, vertices, and asymptotes. Sketch the graph.



Ex 7.

Determine whether the equation represents a parabola, an ellipse, or a hyperbola. If the graph is a parabola, find the vertex, focus, directrix, and focal diameter. If it is an ellipse, find the center, foci, vertices, and lengths of the major and minor axes. If it is a hyperbola, find the center, foci, vertices, and asymptotes. Then sketch the graph of the equation.

$x^2 + 6x + 12y + 9 = 0$