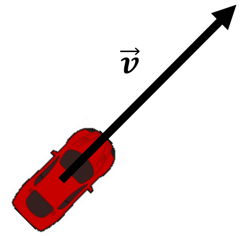
Vectors

(covers parts of Sullivan 9.4)

Suppose a car is heading NE (northeast) at 60 mph.
We can use a vector to help draw a picture (see right).

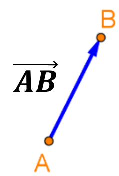


A **vector** consists of two parts:
1. a magnitude (ex: 60 mph)
2. a direction (ex: NE)

By contrast, a **scalar** only has a magnitude (ex: 60 mph).

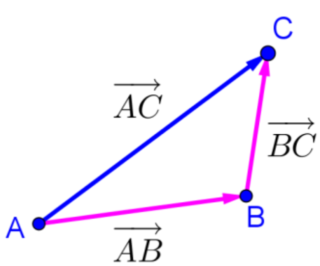
Examples of vectors include: displacement, velocity, acceleration, and force.
Examples of scalars include: distance, speed, time, and volume.

A vector has an **initial point** and a **terminal point**.
The magnitude is the length of the vector.

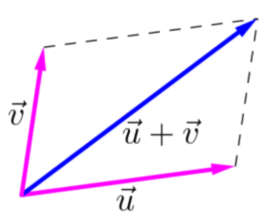


Two vectors are **equal** if they have the same magnitude and direction.
So, all of the vectors to the right are equal.

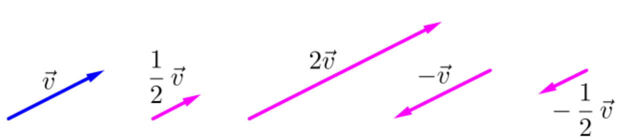
To add two vectors visually, put them tip to tail and make a triangle.



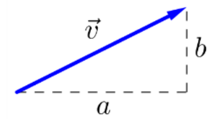
Or put them tail to tail and make a parallelogram.



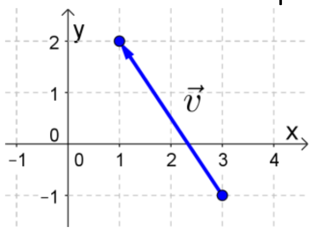
You can stretch and shrink vectors by multiplying by a scalar. Negative values make the vector go in the opposite direction.



In a two-dimensional coordinate system, we can represent vectors using two components: an x-component (horizontal component) and a y-component (vertical component). Here's the notation: $\vec{v} = \langle a, b \rangle$



If vector $\vec{v}$ has initial point $(3, -1)$ and terminal point $(1, 2)$, then $\vec{v} = \langle 1-3,\ 2-(-1) \rangle = \langle -2, 3 \rangle$.



The **magnitude** (or **length**) of a vector $\vec{v} = \langle a, b \rangle$ is $\|\vec{v}\| = \sqrt{\boldsymbol{a^2 + b^2}}$.
ex: The magnitude of $\vec{u} = \langle 2, -3 \rangle$ is $\|\vec{u}\| = \|\langle 2, -3 \rangle\| = \sqrt{2^2 + (-3)^2} = \sqrt{13}$.

The **direction** of a vector $\vec{v} = \langle a, b \rangle$ is θ (in the quadrant that $\vec{v}$ points) where $\boldsymbol{\tan\theta = \frac{b}{a}}$.

Ex 1.
Find the direction (in degrees) of $\vec{u} = \langle -\sqrt{3}, 1 \rangle$.

To add/subtract vectors, you just add/subtract their components.
ex: $\langle 3, -1 \rangle + \langle -4, 7 \rangle = \langle 3 + (-4), -1 + 7 \rangle = \langle -1, 6 \rangle$

To multiply a vector by a scalar, you just "distribute" the scalar to each component.
ex: $-3\langle 2, 5 \rangle = \langle -6, -15 \rangle$

Ex 2.
If $\vec{u} = \langle 2, -3 \rangle$ and $\vec{v} = \langle -1, 2 \rangle$, find $2\vec{u} - 3\vec{v}$.

$\vec{0} = \langle 0, 0 \rangle$ is called the **zero vector**.

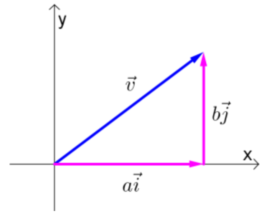
Vectors with length 1 are called ____________________.
ex: $\vec{w} = \langle \frac{3}{5}, \frac{4}{5} \rangle$ is a unit vector since $\|\vec{w}\| = \sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2} = 1$

You can get a unit vector in the direction of any vector $\vec{v}$ by dividing by $\|\vec{v}\|$.
ex: A unit vector in the direction of $\langle -2, 5 \rangle$ would be $\frac{\langle -2,5 \rangle}{\|\langle -2,5 \rangle\|} = \frac{\langle -2,5 \rangle}{\sqrt{(-2)^2 + 5^2}} = \frac{\langle -2,5 \rangle}{\sqrt{29}} = \langle -\frac{2}{\sqrt{29}}, \frac{5}{\sqrt{29}} \rangle$.

Two special unit vectors are given their own letters: $\boldsymbol{\vec{\imath} = \langle 1, 0 \rangle}$ and $\boldsymbol{\vec{\jmath} = \langle 0, 1 \rangle}$

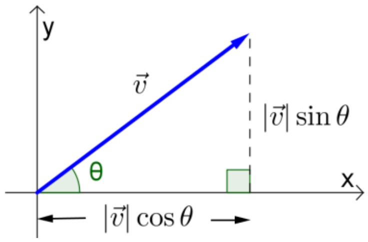
We can represent any vector in terms of $\vec{\imath}$ and $\vec{\jmath}$:
$\vec{v} = \langle a, b\rangle = a\vec{\imath} + b\vec{\jmath}$

ex: $\langle 5, -8\rangle = 5\vec{\imath} - 8\vec{\jmath}$



Ex 3.
If $\vec{u} = 3\vec{\imath} + 2\vec{\jmath}$ and $\vec{v} = -\vec{\imath} + 6\vec{\jmath}$, write $2\vec{u} + 5\vec{v}$ in terms of $\vec{\imath}$ and $\vec{\jmath}$.

It is often useful to break a vector into its horizontal and vertical components.



Ex 4.
An airplane heads due north with an airspeed of 300 mi/h (this is speed relative to the air). It experiences a 40 mi/h crosswind blowing in the direction $N\ 30°\ E$. Express the velocity $\vec{v}$ of the airplane relative to the air, and the velocity $\vec{u}$ of the wind, in component form.

Find the velocity of the airplane relative to the ground.

Find the speed and direction of the airplane's movement relative to the ground.

Ex 5.
A woman launches a boat from one shore of a straight river and wants to land at the point directly on the opposite shore. If the speed of the boat (relative to the water) is 10 mi/h and the river is flowing east at the rate of 5 mi/h, in what direction should she head the boat in order to arrive at the desired landing point?

Ex 6.
A 600-pound calculus book is suspended from two ropes. Find the tension in each rope. (Note: Since the book is not moving, it said to be in **static equilibrium**, so the sum of all forces acting on it is zero.)