Trigonometric Identities

(covers parts of Sullivan 7.4, 7.5, and 7.6)

Here are some useful trig identities. (☺ = memorize)

Reciprocal Identities

$\tan x = \frac{\sin x}{\cos x}$ ☺ $\quad \cot x = \frac{\cos x}{\sin x}$ ☺

$\csc x = \frac{1}{\sin x}$ ☺ $\quad \sec x = \frac{1}{\cos x}$ ☺ $\quad \cot x = \frac{1}{\tan x}$ ☺

Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$ ☺ $\quad \tan^2 x + 1 = \sec^2 x$ ☺ $\quad 1 + \cot^2 x = \csc^2 x$ ☺

Even-Odd Identities

$\sin(-x) = -\sin x$ ☺ $\quad \cos(-x) = \cos x$ ☺ $\quad \tan(-x) = -\tan x$ ☺

$\csc(-x) = -\csc x \quad \sec(-x) = \sec x \quad \cot(-x) = -\cot x$

Cofunction Identities

$\sin\left(\frac{\pi}{2} - x\right) = \cos x \quad \cos\left(\frac{\pi}{2} - x\right) = \sin x \quad \tan\left(\frac{\pi}{2} - x\right) = \cot x$

$\csc\left(\frac{\pi}{2} - x\right) = \sec x \quad \sec\left(\frac{\pi}{2} - x\right) = \csc x \quad \cot\left(\frac{\pi}{2} - x\right) = \tan x$

Addition and Subtraction Formulas

$\sin(x + y) = \sin x \cos y + \cos x \sin y$ ☺

$\sin(x - y) = \sin x \cos y - \cos x \sin y$ ☺

$\cos(x + y) = \cos x \cos y - \sin x \sin y$ ☺

$\cos(x - y) = \cos x \cos y + \sin x \sin y$ ☺

$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$

$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

Double-Angle Formulas

$\sin 2x = 2 \sin x \cos x$ ☺

$\cos 2x = \cos^2 x - \sin^2 x$ ☺

$= 1 - 2\sin^2 x$ ☺

$= 2\cos^2 x - 1$ ☺

$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$

Power-Lowering Formulas

$\sin^2 x = \frac{1 - \cos 2x}{2}$ ☺

$\cos^2 x = \frac{1 + \cos 2x}{2}$ ☺

$\tan^2 x = \frac{1 - \cos 2x}{1 + \cos 2x}$

Half-Angle Formulas

$\sin\frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{2}}$ ☺

$\cos\frac{x}{2} = \pm\sqrt{\frac{1 + \cos x}{2}}$ ☺

$\tan\frac{x}{2} = \frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x}$

There are two main ways to prove identities:

1. Start from one side (usually the more complicated side), and take steps to get to the other side.

OR

2. Simplify both sides until they look the same.

If you get stuck, convert to sin and cos.

Ex 1.

Prove the identity.

$\sec x + \tan x = \frac{\cos x}{1-\sin x}$

Ex 2.

Prove the identity.

$2 \tan x \sec x = \frac{1}{1-\sin x} - \frac{1}{1+\sin x}$

Ex 3.

Prove the identity.

$\frac{1+\cos\theta}{\cos\theta} = \frac{\tan^2\theta}{\sec\theta-1}$

Ex 4.

Find the exact value of $\sin\left(2\cos^{-1}\left(-\frac{3}{5}\right)\right)$ without a calculator.

Ex 5.

Write $\sin(\cos^{-1}x + \tan^{-1}y)$ as an algebraic expression in x and y, where $-1 \leq x \leq 1$ and y is any real number.

Ex 6.

Suppose $\cos x = -\frac{2}{3}$ and x is in Quadrant II. Find $\cos 2x$ and $\sin 2x$.

Ex 7.

Suppose $\sin x = \frac{2}{5}$ and x is in Quadrant II. Find $\tan \frac{x}{2}$.

Ex 8.
Express $\sin^2 x \cos^2 x$ in terms of the first power of cosine.