

1. Prove the identity: $\frac{1-\cos x}{\sin x} + \frac{\sin x}{1-\cos x} = 2 \csc x$

2. Prove the identity: $1 - \tan x \tan y = \frac{\cos(x+y)}{\cos x \cos y}$

3. Write $\cos(2 \sin^{-1} x)$ as an algebraic expression in x .

4. Suppose $\cot x = 5$ and $180^\circ < x < 270^\circ$. Find $\sin \frac{x}{2}$.

5. Express $\cos^4 x$ in terms of the first power of cosine. (Hint: Start by writing $\cos^4 x = (\cos^2 x)^2$.)

Q: A man leaves home and, after making three left turns, he ends up back at home, and finds two masked men waiting for him. What is happening?

6. Prove the identity: $\frac{\csc^2 \theta}{1 + \tan^2 \theta} = \cot^2 \theta$

7. Prove the identity: $\sin x + \cos x \cot x = \csc x$

8. Prove the identity: $\frac{\cot x}{\csc x + 1} = \frac{\csc x - 1}{\cot x}$

9. Prove the identity: $\frac{1 + \sin x}{1 - \sin x} - \frac{1 - \sin x}{1 + \sin x} = 4 \tan x \sec x$

10. Prove the identity: $\csc^4 \alpha - \cot^4 \alpha = \cot^2 \alpha + \csc^2 \alpha$

11. Prove the identity: $\frac{1}{\tan \theta + \cot \theta} = \sin \theta \cos \theta$

12. Prove the identity: $\tan x + \tan y = \frac{\sin(x+y)}{\cos x \cos y}$

13. Prove the identity: $\tan\left(x + \frac{\pi}{4}\right) = \frac{1+\tan x}{1-\tan x}$

14. Prove the identity: $\cos^4 x - \sin^4 x = \cos 2x$

15. Prove the identity: $\csc 2x = \frac{1}{2}\sec x \csc x$

16. Find the exact value of $\sin \left[\tan^{-1} \frac{3}{4} - \cos^{-1} \left(-\frac{4}{5} \right) \right]$ without a calculator.

17. Find the exact value of $\tan \left(\tan^{-1} \frac{4}{3} + \cos^{-1} \frac{8}{17} \right)$ without a calculator.

18. Write $\cos(\sin^{-1} x + \tan^{-1} y)$ as an algebraic expression in x and y , where $-1 \leq x \leq 1$ and y is any real number.

19. Write $\sin(\tan^{-1} x - \sin^{-1} y)$ as an algebraic expression in x and y , where x is any real number $-1 \leq y \leq 1$.

20. Find the exact value of $\cos\left(2 \sin^{-1} \frac{4}{5}\right)$ without a calculator.

21. Find the exact value of $\sin\left(2 \tan^{-1}\left(-\frac{3}{2}\right)\right)$ without a calculator.

22. Write $\cos(2 \tan^{-1} x)$ as an algebraic expression in x only.

23. Write $\tan(2 \sin^{-1} x)$ as an algebraic expression in x only, where $-1 \leq x \leq 1$.

24. Suppose $\sin x = -\frac{5}{7}$ and $180^\circ < x < 270^\circ$. Find $\cos 2x$, $\sin 2x$, and $\sin \frac{x}{2}$.

25. Suppose $\sec x = \frac{5}{2}$ and $270^\circ < x < 360^\circ$. Find $\tan 2x$, $\cos \frac{x}{2}$, and $\tan \frac{x}{2}$.

26. Suppose $\csc x = \frac{3}{2}$ and $\cos x > 0$. Find $\sin 2x$, $\tan 2x$, and $\cos \frac{x}{2}$.

27. Suppose $\tan x = -\frac{1}{6}$ and $\sin x > 0$. Find $\cos 2x$, $\sin 2x$, and $\sin \frac{x}{2}$.

28. Express $\sin^4 x$ in terms of the first power of cosine.

29. Express $\sin^2 x \cos^2 x$ in terms of the first power of cosine.

Optional exercises from the Sullivan book if you'd like more practice:

7.4 (p.475) #21-85 odd

7.5 (p.487) #49-69 odd, 75-91 odd

7.6 (p.497) #9-19 odd, 49-57 odd, 81-91 odd