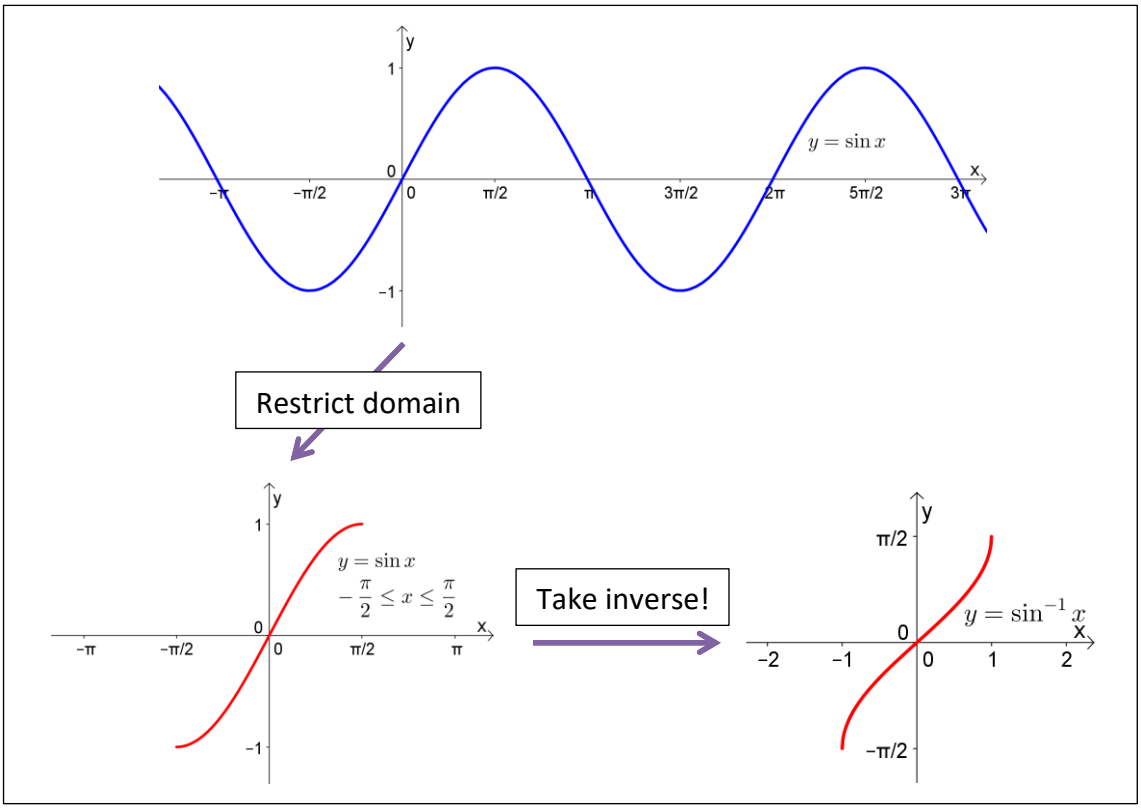
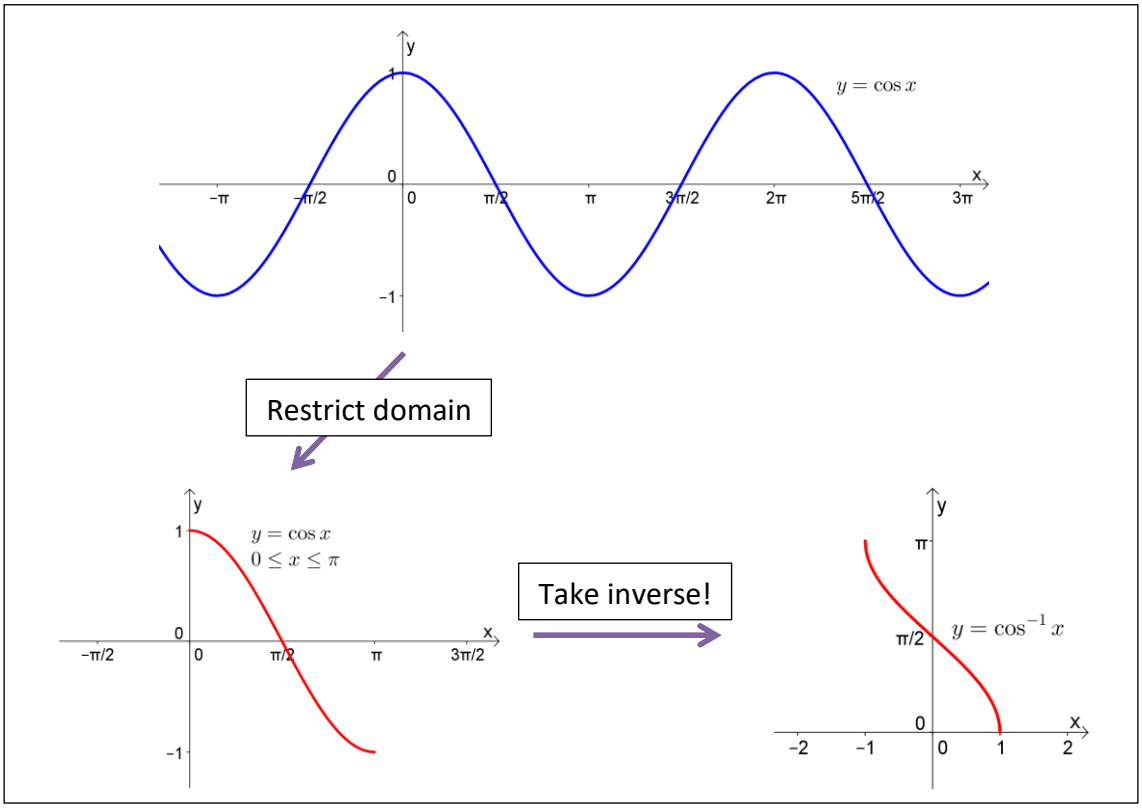
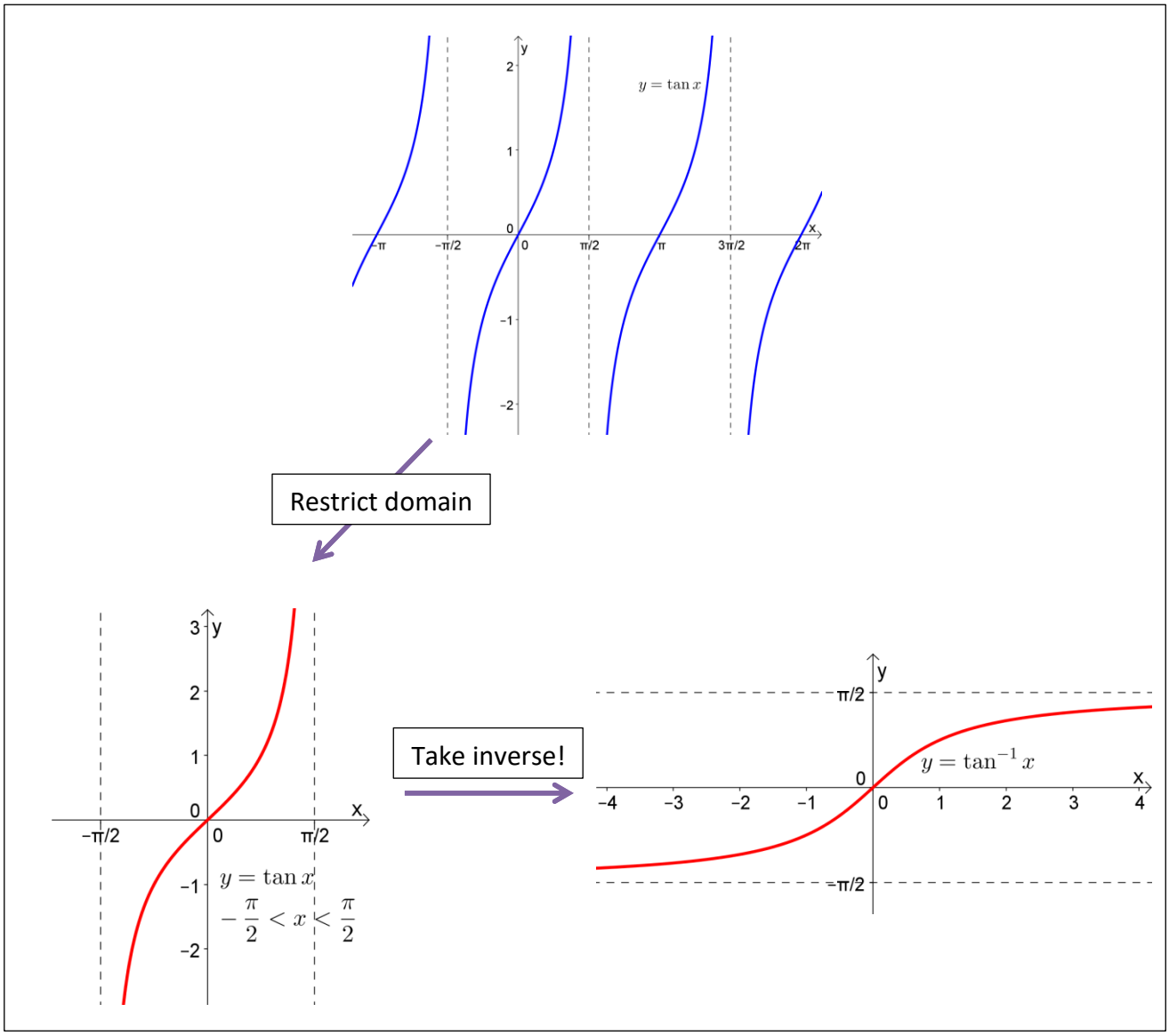
Inverse Trigonometric Functions

(covers Sullivan 7.1 and 7.2)

Only one-to-one functions have inverses. So, how can we make inverses of the trig functions? We first need to restrict their domains to make them one-to-one.

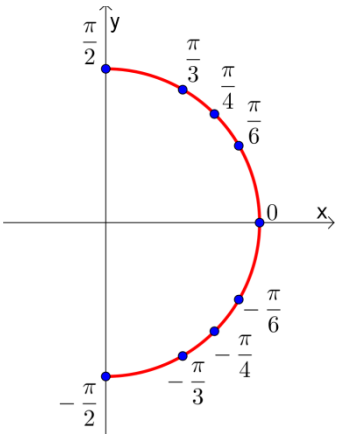




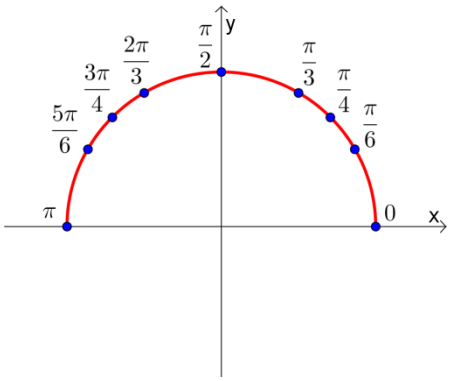


	Domain	Range
$\sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1} x$	$(-\infty, \infty)$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\sin^{-1} x$ and
$\tan^{-1} x$ output:



$\cos^{-1} x$ outputs:



Ex 1.
Find each value.

$\sin^{-1}\frac{1}{2} =$

$\sin^{-1}\left(-\frac{1}{2}\right) =$

$\sin^{-1}\frac{3}{2}$

$\sin^{-1}\left(\sin\frac{\pi}{3}\right) =$

$\sin^{-1}\left(\sin\frac{2\pi}{3}\right) =$

$\cos^{-1}\frac{\sqrt{3}}{2} =$

$\cos^{-1} 0 =$

$\cos^{-1}\left(\cos\frac{2\pi}{3}\right) =$

$\cos^{-1}\left(\cos\frac{5\pi}{3}\right) =$

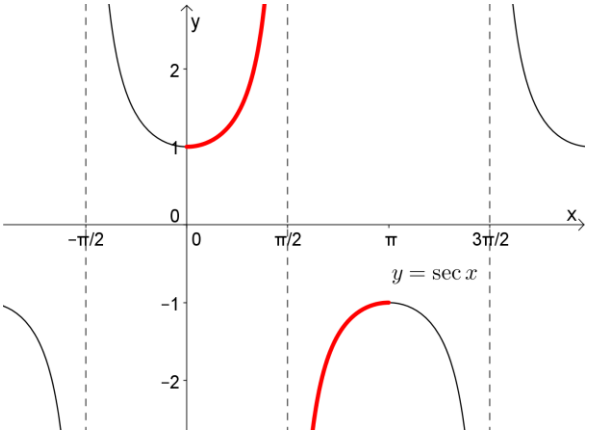
$\tan^{-1} 1 =$

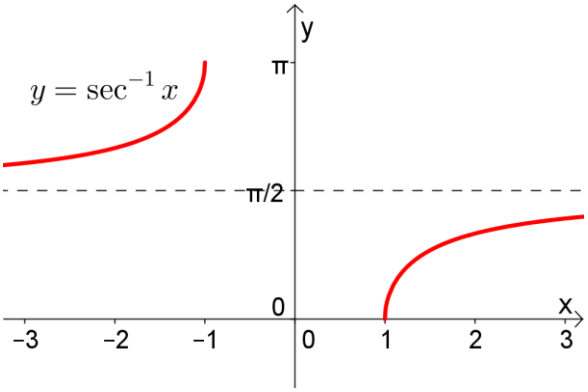
$\tan^{-1}\sqrt{3} =$

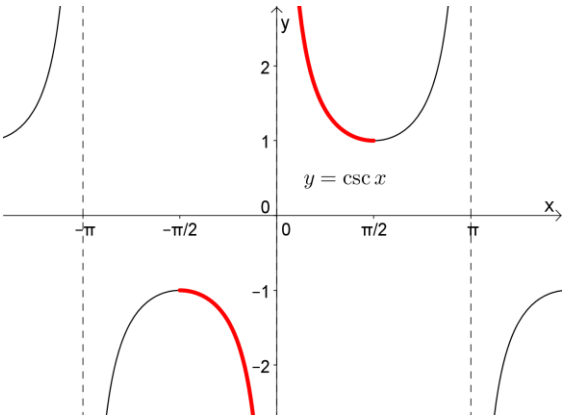
Ex 2.
Find the exact value of the expression.
$\tan\left(\sin^{-1}\frac{12}{13}\right)$

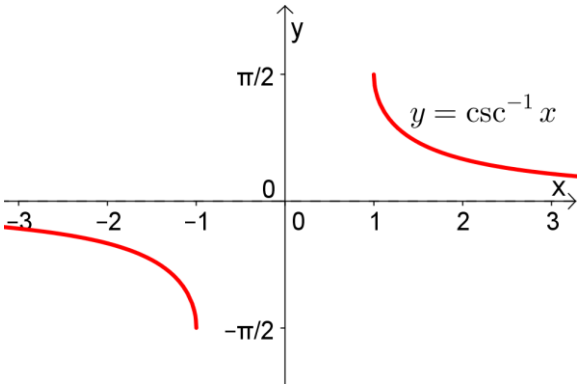
Ex 3.
Rewrite the expression as an algebraic expression in x.
$\cos(\tan^{-1}x)$

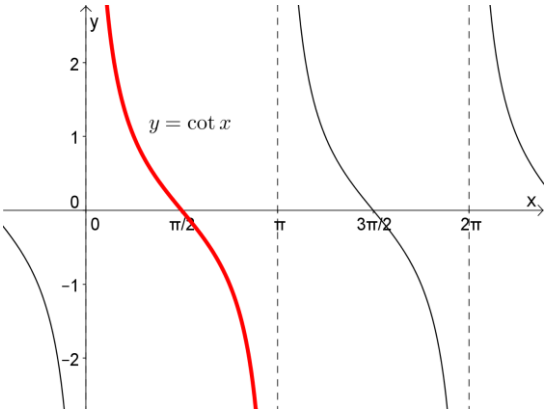
Here are the other inverse trig functions:

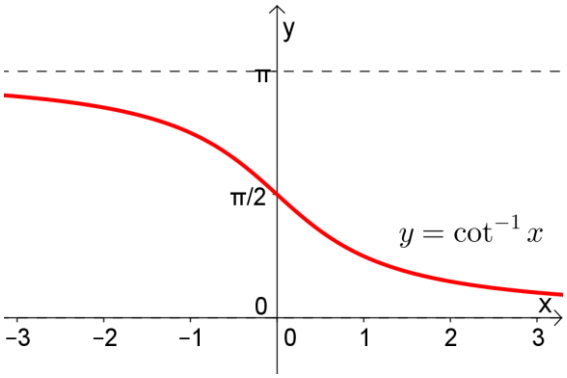





y
x
2
1
0
−1
−2
−π
−π/2
0
π/2
π
$y = \csc x$


y
x
π/2
0
−π/2
−3
−2
−1
0
1
2
3
$y = \csc^{-1} x$


y
x
2
1
0
−1
−2
0
π/2
π
3π/2
2π
$y = \cot x$


y
x
π
π/2
0
−3
−2
−1
0
1
2
3
$y = \cot^{-1} x$