Homework Solutions for Packet #20

1. Find each value.

$\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \boxed{-\frac{\pi}{4}}$

$\sin \square = -\frac{\sqrt{2}}{2}$, $\square \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\cos^{-1} 1 = \boxed{0}$

$\cos \square = 1$, $\square \in [0, \pi]$

$\tan^{-1}(-1) = \boxed{-\frac{\pi}{4}}$

$\tan \square = -1$, $\square \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\tan^{-1} \frac{\sqrt{3}}{3} = \boxed{\frac{\pi}{6}}$

$\tan \square = \frac{\sqrt{3}}{3}$, $\square \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\sin^{-1}\left(\sin \frac{5\pi}{4}\right) = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \boxed{-\frac{\pi}{4}}$

$\sin \square = -\frac{\sqrt{2}}{2}$, $\square \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\cos^{-1}(\cos \pi) = \cos^{-1}(-1) = \boxed{\pi}$

$\cos \square = -1$, $\square \in [0, \pi]$

$\cos^{-1}\left(\cos \frac{7\pi}{2}\right) = \cos^{-1}(0) = \boxed{\frac{\pi}{2}}$

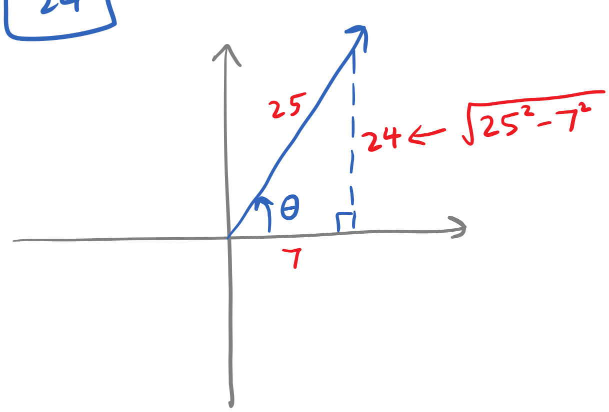
$\cos \square = 0$, $\square \in [0, \pi]$

2. Find the exact value of the expression.

$\csc\left(\underbrace{\cos^{-1} \frac{7}{25}}_{\theta}\right) = \csc \theta = \boxed{\frac{25}{24}}$

$\cos^{-1} \frac{7}{25} = \theta \leftarrow [0, \pi]$

$\cos \theta = \frac{7}{25}$

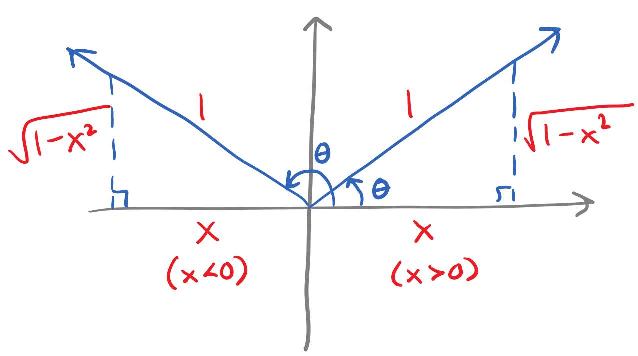


3. Rewrite the expression as an algebraic expression in x.

$\tan(\underbrace{\cos^{-1} x}_{\theta}) = \tan\theta = \boxed{\dfrac{\sqrt{1-x^2}}{x}}$

$\cos^{-1} x = \theta \leftarrow [0, \pi]$

$\cos\theta = x = \dfrac{x}{1}$



Q: A man rode his horse into town on Tuesday. Two days later he rode home on Tuesday. How is this possible?

4. Find the exact value in radians without a calculator.

a) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \boxed{-\frac{\pi}{3}}$

$\sin \square = -\frac{\sqrt{3}}{2}$

$\uparrow$

$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

c) $\sin^{-1}\frac{\pi}{2}$ $\boxed{\text{undefined}}$

$\frac{\pi}{2} > 1$

Input to $\sin^{-1} x$ is $[-1, 1]$

e) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \boxed{\frac{3\pi}{4}}$

$\cos \square = -\frac{\sqrt{2}}{2}$

$\uparrow$

$[0, \pi]$

g) $\tan^{-1}\sqrt{3} = \boxed{\frac{\pi}{3}}$

$\tan \square = \sqrt{3}$

$\uparrow$

$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

i) $\sec^{-1}\left(-\frac{2\sqrt{3}}{3}\right) = \boxed{\frac{5\pi}{6}}$

$\sec \square = \frac{-2\sqrt{3}}{3}$

$\uparrow$

$[0, \pi]$

$\downarrow$

$\cos \square = -\frac{3\cdot\sqrt{3}}{2\sqrt{3}\cdot\sqrt{3}} = \frac{-\cancel{3}\sqrt{3}}{2\cdot\cancel{3}} = -\frac{\sqrt{3}}{2}$

5. Find the exact value without a calculator.

a) $\sin^{-1}\left(\sin\frac{\pi}{3}\right)$

$= \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$= \boxed{\frac{\pi}{3}}$

$\sin \square = \frac{\sqrt{3}}{2}$

$\uparrow$

$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

c) $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$

$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$= \boxed{\frac{\pi}{6}}$

$\tan \square = \frac{1}{\sqrt{3}}$

$\uparrow$

$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

e) $\tan(\cos^{-1} 0)$

$= \tan \frac{\pi}{2}$

$\boxed{\text{undefined}}$

$\cos \square = 0$

$\uparrow$

$[0, \pi]$

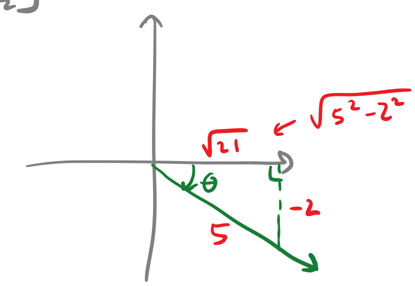
g) $\cot\left(\underbrace{\sin^{-1}\left(-\frac{2}{5}\right)}_{\theta}\right)$

$\sin^{-1}\left(-\frac{2}{5}\right) = \theta \leftarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\sin\theta = \frac{-2}{5}$

$= \cot\theta$

$= \boxed{-\frac{\sqrt{21}}{2}}$



i) $\csc\left(\underbrace{\cos^{-1}\frac{1}{5}}_{\theta}\right)$

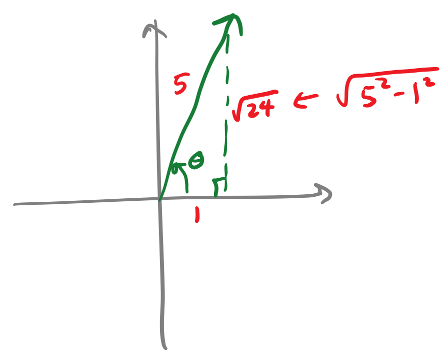
$\cos^{-1}\frac{1}{5} = \theta \leftarrow [0, \pi]$

$\cos\theta = \frac{1}{5}$

$= \csc\theta$

$= \frac{5}{\sqrt{24}}$

$= \boxed{\frac{5}{2\sqrt{6}}}$



6. Rewrite each expression as an algebraic expression in x.

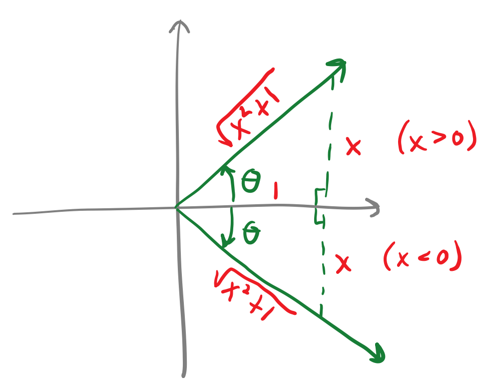
a) $\sin(\underbrace{\tan^{-1} x}_{\theta})$

$\tan^{-1} x = \theta \leftarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\tan\theta = x = \frac{x}{1}$

$= \sin\theta$

$= \boxed{\frac{x}{\sqrt{x^2+1}}}$



c) $\cos(\underbrace{\sin^{-1} x}_{\theta})$

$$\sin^{-1} x = \theta \leftarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\sin\theta = x = \frac{x}{1}$$

$$= \cos\theta$$

$$= \frac{\sqrt{1-x^2}}{1}$$

$$= \boxed{\sqrt{1-x^2}}$$