Trigonometric Functions (using Triangles)

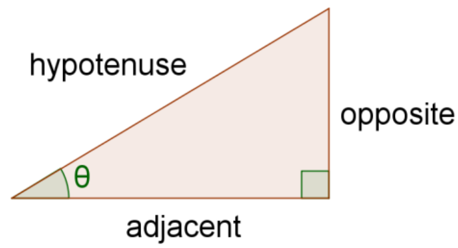
(covers Sullivan 8.1)

Recall: To convert 72° into radians: $72^\circ \times \frac{\boldsymbol{\pi}}{\mathbf{180^\circ}} = \frac{2\pi}{5}$. To convert $\frac{\pi}{3}$ into degrees: $\frac{\pi}{3} \times \frac{\mathbf{180^\circ}}{\boldsymbol{\pi}} = 60^\circ$.

All of the trig functions can also be defined using triangles.

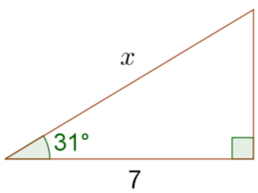
$\sin\theta = \frac{\text{opp}}{\text{hyp}}$ $\qquad \cos\theta = \frac{\text{adj}}{\text{hyp}}$ $\qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$

$\csc\theta = \frac{\text{hyp}}{\text{opp}}$ $\qquad \sec\theta = \frac{\text{hyp}}{\text{adj}}$ $\qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$



(opp = opposite, adj = adjacent, hyp = hypotenuse)

Ex 1.
Find x.



Ex 2.
Sketch a triangle that has acute angle θ, and find the other five trigonometric ratios of θ.

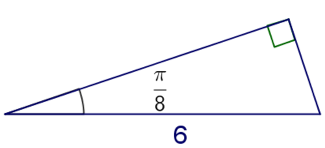
$\sin\theta = \frac{4}{5}$

$\csc\theta =$ ________

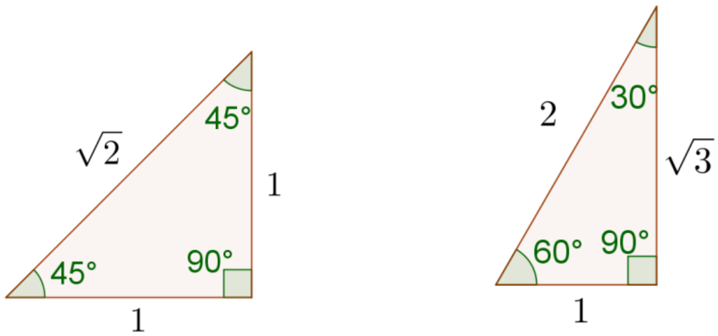
$\cos\theta =$ ________ $\qquad \sec\theta =$ ________

$\tan\theta =$ ________ $\qquad \cot\theta =$ ________

Ex 3.
Solve the right triangle.

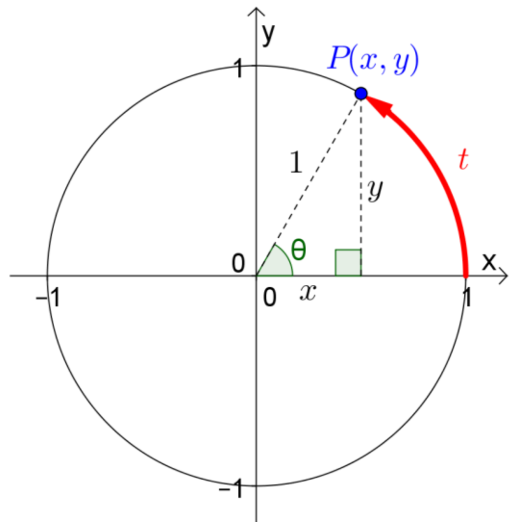


Let's take a moment to remember our buddies, the special triangles (45°-45°-90° and 30°-60°-90°).

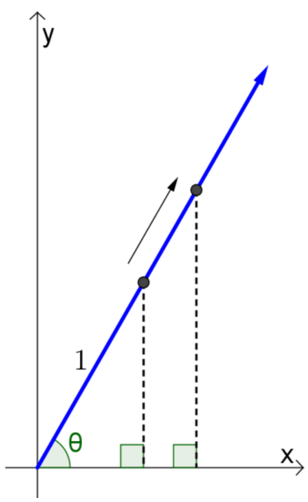


To the right is a picture that relates "unit circle" trig with "triangle" trig. Note that if θ is in radians, then $\theta = t$.

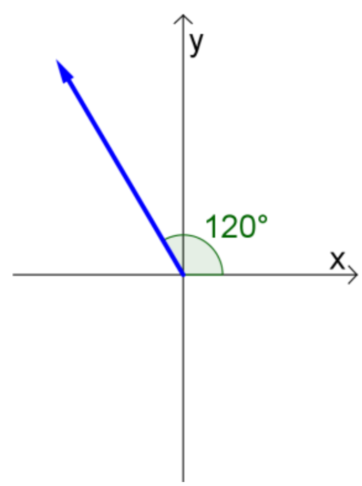
Here, we still have $\sin t = y$ and $\cos t = x$. Why?
Because $\sin t = \sin \theta = \frac{y}{1} = y$ and $\cos t = \cos \theta = \frac{x}{1} = x$.



Since right triangles that share an angle θ are similar, we don't have to be restricted to the unit circle (see below).



For large θ angles, we use reference angles ($\bar{\theta}$). (see right)



Ex 4.
Find the exact value of each trigonometric function.
$\sin 240°$

$\cot 495°$

Ex 5.
If $\tan\theta = \frac{2}{3}$ and $\sin\theta < 0$, find $\cos\theta$.

Here are a few more formulas from your trig class for reference:
$\boldsymbol{s = \theta r}$ (this gives you arc length s made by an angle θ in a circle of radius r)
$\boldsymbol{A = \frac{1}{2} r^2 \theta}$ (this gives you area A of a sector with central angle θ in a circle of radius r)
$\boldsymbol{A = \frac{1}{2} ab \sin\theta}$ (this gives you area A of a triangle with sides a and b that make an angle θ)