Modeling with Exponential Functions

(covers Sullivan 5.8)

Uninhibited exponential growth can be modeled by the following function:
$\boldsymbol{N(t) = N_0 e^{kt}}$

$N(t)$ = population at time t
N_0 = initial population (when $t = 0$)
k = relative growth rate (ex: 0.02 would represent 2% of the population at time t)
t = time

Ex 1.
You initially see 500 bacteria in a petri dish. An hour later, you see 800 bacteria in the petri dish! Assuming the bacteria population grows exponentially, how many will be in the petri dish after 5 hours after your initial count?

When will there be 20,000 bacteria in the petri dish?

Newton's Law of Cooling

The rate at which an object cools is proportional to the temperature difference between the object and its surroundings. From this (and using calculus), we get the following model:

$$\boldsymbol{T(t) = T_s + (T_0 - T_s)e^{-kt}}$$

$T(t)$ = temperature at time t
T_s = surrounding temperature
T_0 = initial temperature of object
k = positive constant that depends on type of object
t = time

Ex 2.

Ethiopian coffee sits in your newly-bought Mt. SAC alumni mug at an initial temperature of $200°\text{F}$. The room temperature is $70°\text{F}$. After 10 minutes, the temperature of the coffee is $150°\text{F}$. Find a function that models the temperature of the coffee at time t (in minutes).

Find the temperature of the coffee after 15 minutes.

When will the coffee be $100°\text{F}$?