1. Expand using the properties of logarithms: $\ln\left(\sqrt{\frac{x(x-1)}{x^2+2}}\right)$ (Hint: first bring down the ½ exponent)

$$\ln\left(\frac{x(x-1)}{x^2+2}\right)^{1/2} = \frac{1}{2}\ln\frac{x(x-1)}{x^2+2}$$
$$= \boxed{\frac{1}{2}\left[\ln x + \ln(x-1) - \ln(x^2+2)\right]}$$

2. Write as a single logarithm and simplify:
$\frac{1}{2}\log_5(2x+1) + \log_5(x^2-1) - \log_5(x-1) - 3\log_5(x-4)$

$$= \log_5(2x+1)^{1/2} + \log_5(x^2-1) - \log_5(x-1) - \log_5(x-4)^3$$
$$= \log_5\frac{(2x+1)^{1/2}(x^2-1)}{(x-1)(x-4)^3}$$
$$= \log_5\frac{(2x+1)^{1/2}(x+1)\cancel{(x-1)}}{\cancel{(x-1)}(x-4)^3} = \boxed{\log_5\frac{(x+1)\sqrt{2x+1}}{(x-4)^3}}$$

3. Evaluate without a calculator: $\log_5 105 - \log_5 3 - \log_5 7$

$$= \log_5\frac{105}{(3)(7)} = \log_5 5 = \boxed{1}$$

4. Use the Change of Base Theorem to rewrite $\log_2 0.3$ in terms of common logarithms. Then use a calculator to evaluate to four decimal places.

$$\log_2 0.3 = \boxed{\frac{\log 0.3}{\log 2}} \approx \boxed{-1.7370}$$

Challenge: Prove that $\log_a A + \log_a B = \log_a(AB)$. (Hint: Start from the LHS and rewrite as $\log_a\left(a^{\log_a A + \log_a B}\right)$.)

$$\log_a A + \log_a B = \log_a\left(a^{\log_a A + \log_a B}\right)$$
$$= \log_a\left(a^{\log_a A}\, a^{\log_a B}\right)$$
$$= \log_a(AB)$$

$\blacksquare$

Q: How can half of 12 be 7?

5. Expand using the properties of logarithms: $\log_3\left(\frac{x^3}{3(y-1)(z+2)^2}\right)$

$$= \log_3 x^3 - \log_3 3 - \log_3 (y-1) - \log_3 (z+2)^2$$

$$= \boxed{3\log_3 x - 1 - \log_3 (y-1) - 2\log_3 (z+2)}$$

7. Expand using the properties of logarithms: $\log\sqrt{\frac{10x^2-10x-60}{(x+1)^7}}$ ← $10(x^2-x-6)$ $10(x+2)(x-3)$

$$= \log\left(\frac{10(x+2)(x-3)}{(x+1)^7}\right)^{1/2}$$

$$= \frac{1}{2}\log\left(\frac{10(x+2)(x-3)}{(x+1)^7}\right)$$

$$= \frac{1}{2}\left[\log 10 + \log(x+2) + \log(x-3) - \log(x+1)^7\right]$$

$$= \boxed{\frac{1}{2}\left[1 + \log(x+2) + \log(x-3) - 7\log(x+1)\right]}$$

9. Write as a single logarithm and simplify: $\ln(x+1) + 2\ln x - \frac{1}{2}\ln(3x-1) - 5\ln(7-x)$

$$= \ln(x+1) + \ln x^2 - \ln(3x-1)^{1/2} - \ln(7-x)^5$$

$$= \ln \frac{(x+1)x^2}{(3x-1)^{1/2}(7-x)^5}$$

$$= \boxed{\ln \frac{x^2(x+1)}{\sqrt{3x-1}\,(7-x)^5}}$$

11. Write as a single logarithm and simplify: $2\ln(x-1) - \frac{1}{2}\ln(x^2+1) - 3\ln(x^2-1)$

$$= \ln(x-1)^2 - \ln(x^2+1)^{1/2} - \ln(x^2-1)^3$$

$$= \ln \frac{(x-1)^2}{(x^2+1)^{1/2}(x^2-1)^3}$$

$$= \ln \frac{(x-1)^2}{\sqrt{x^2+1}\,[(x+1)(x-1)]^3}$$

$$= \ln \frac{\cancel{(x-1)^2}}{\sqrt{x^2+1}\,(x+1)^3(x-1)^{\cancel{3}\,1}} = \boxed{\ln \frac{1}{(x+1)^3(x-1)\sqrt{x^2+1}}}$$

13. Write as a single logarithm and simplify:
$\frac{1}{5}\log_4(1-3x) - 2\log_4(x+3) + \log_4(x^2-9) - \frac{1}{2}\log_4(x^2+4)$

$$= \log_4 (1-3x)^{1/5} - \log_4 (x+3)^2 + \log_4 (x^2-9) - \log (x^2+4)^{1/2}$$

$$= \log_4 \frac{(1-3x)^{1/5}(x^2-9)}{(x+3)^2 (x^2+4)^{1/2}}$$

$$= \log_4 \frac{\cancel{(x+3)}(x-3)\sqrt[5]{1-3x}}{(x+3)^{\cancel{2}\,1}\sqrt{x^2+4}}$$

$$= \boxed{\log_4 \frac{(x-3)\sqrt[5]{1-3x}}{(x+3)\sqrt{x^2+4}}}$$

15. Evaluate without a calculator: $7^{\log_7 3 + \log_7 4}$

$$= 7^{\log_7 12}$$ $3 \cdot 4$

$$= \boxed{12}$$

$$a^{\log_a x} = x$$

17. Use the Change of Base Theorem to rewrite $\log_2 e$ in terms of natural logarithms. Then use a calculator to evaluate to four decimal places.

$$\log_2 e = \boxed{\frac{\ln e}{\ln 2}} \approx \boxed{1.4427}$$