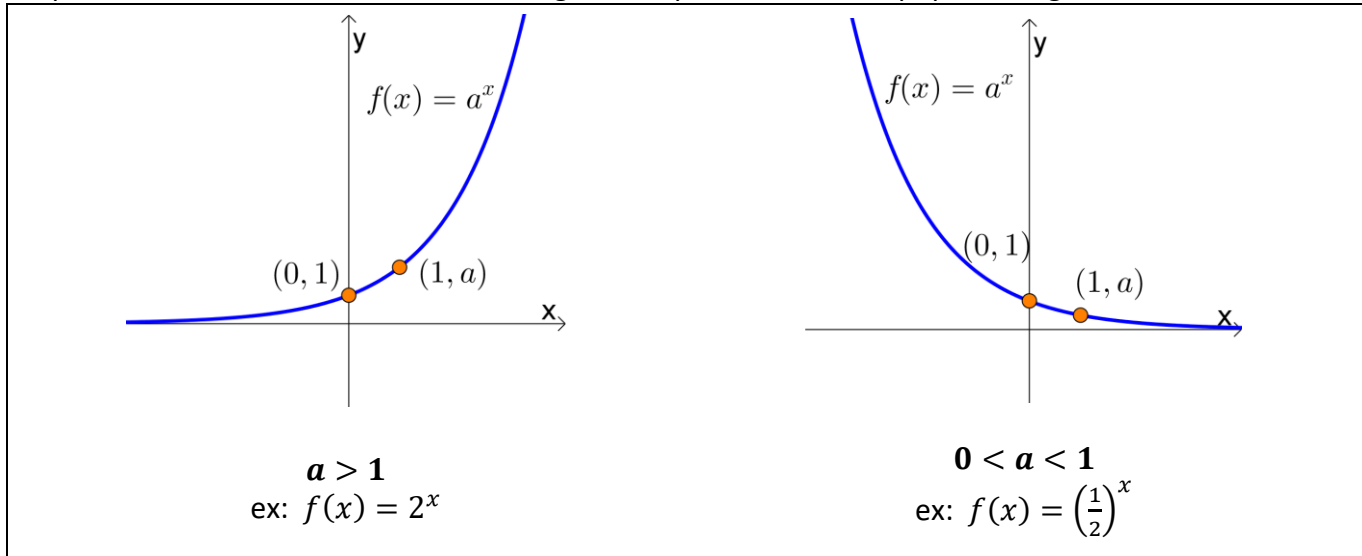


Exponential and Logarithmic Functions

(covers Sullivan 5.3, 5.4)

Exponential functions are useful modeling certain phenomena, like population growth.



Note: a is called the base of the exponential function. $a > 0$ and $a \neq 1$

What is the horizontal asymptote? _____

What is the domain? _____

What is the range? _____

Natural Base e

e (called the natural base) is defined to be the number that $\left(1 + \frac{1}{n}\right)^n$ approaches as n gets larger and larger. From the compound interest formula $A = P \left(1 + \frac{r}{n}\right)^{nt}$, we can interpret this as investing \$1 at a rate of 100% per year for 1 year, compounded n times per year.

n	$\left(1 + \frac{1}{n}\right)^n$
1	2
2	2.25
5	2.48832
10	2.59374
100	2.70481
1,000	2.71692
1,000,000	2.71828

Continuing, we get that e is the irrational number 2.718281828459045...

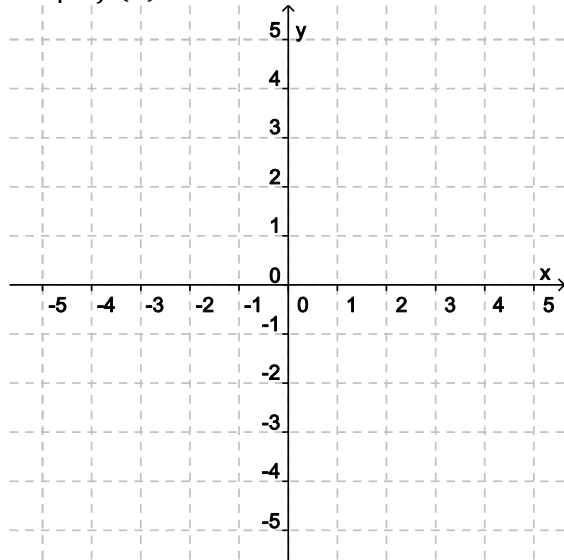
Notes:

The graph of e^x is between the graphs of 2^x and 3^x . (Since $2 < e < 3$.)

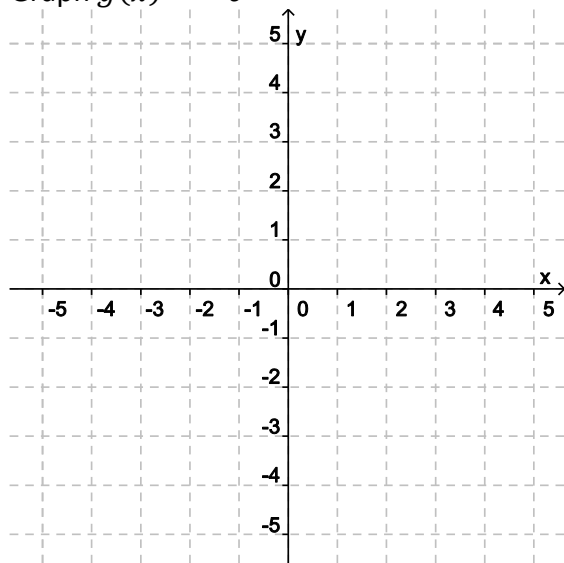
In calculus, you learn that e^x has some amazing properties...

Ex 1.

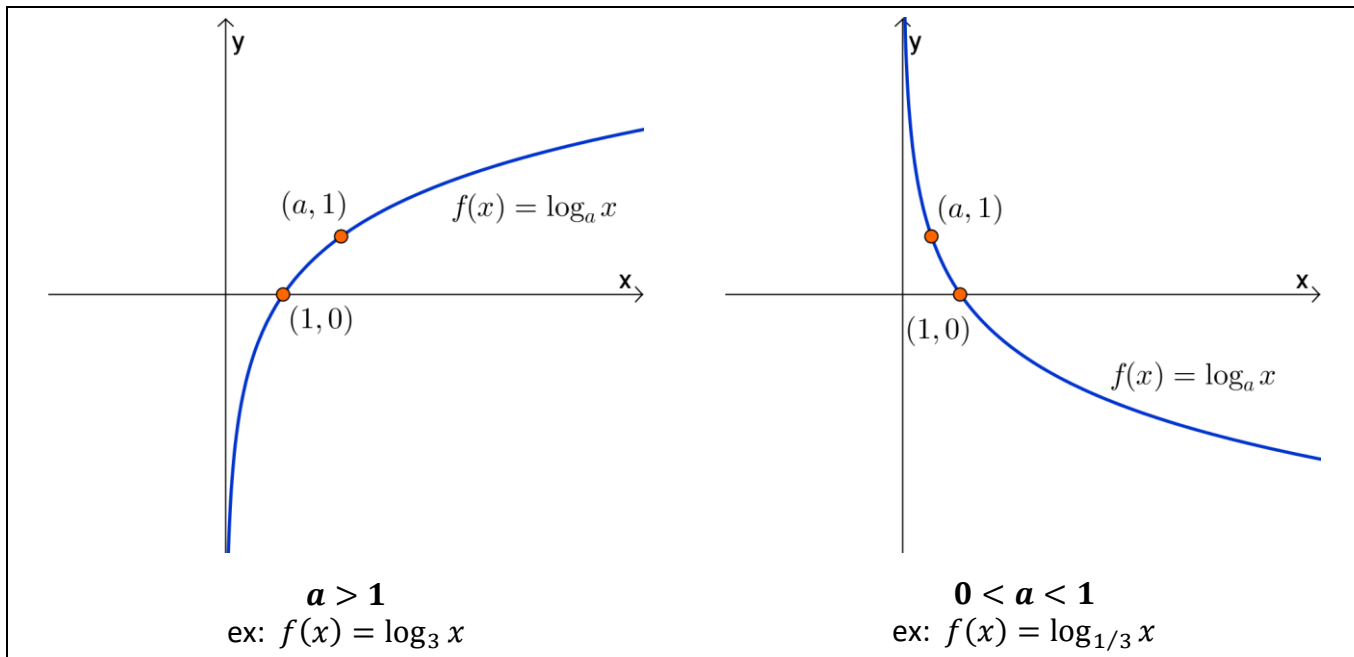
Graph $f(x) = 2^{x-2} + 1$

**Ex 2.**

Graph $g(x) = -e^{-x}$



One-to-one functions have inverses. The inverse of an exponential function is a logarithmic function. In general, the **logarithmic function with base a** is $f(x) = \log_a x$ (where $a > 0, a \neq 1, x > 0$).



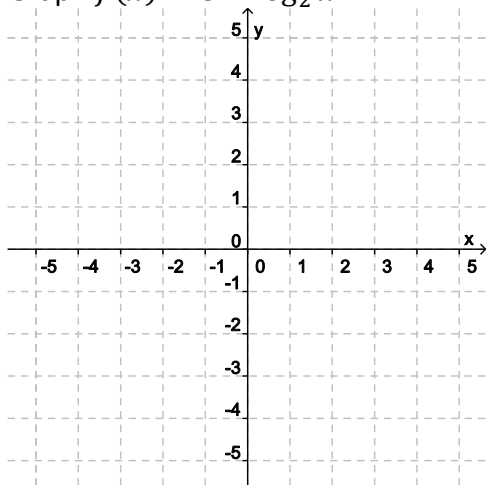
What is the vertical asymptote? _____

What is the domain? _____

What is the range? _____

Ex 3.

Graph $f(x) = 3 - \log_2 x$



Note:

$\log x = \log_{10} x$ (_____ logarithm)

$\ln x = \log_e x$ (_____ logarithm)

Ex 4.

Evaluate.

$$\log_2 8 =$$

$$\log_3 3 =$$

$$\log_e \frac{1}{e^3} =$$

$$\log_{36} 6 =$$

$$\ln e^3 =$$

$$\log 1000 =$$

Note: $\log_a a = \underline{\hspace{1cm}}$ and $\log_a 1 = \underline{\hspace{1cm}}$

Note: Since a^x and $\log_a x$ are inverse functions by definition, $a^{\log_a x} = \underline{\hspace{1cm}}$ and $\log_a a^x = \underline{\hspace{1cm}}$
(ex: $\log_7 7^8 = \underline{\hspace{1cm}}$ $3^{\log_3 7} = \underline{\hspace{1cm}}$)