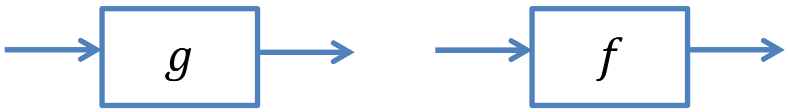
Composite, One-to-one, and Inverse Functions

(covers Sullivan 5.1, 5.2)

Another way to combine two functions $f(x)$ and $g(x)$ is to plug one into the other.
This is called ______________________________.

The **composite function** $f \circ g$ is defined by $(\boldsymbol{f} \circ \boldsymbol{g})(\boldsymbol{x}) = \boldsymbol{f}(\boldsymbol{g}(\boldsymbol{x}))$.



The domain of $f \circ g$ is the set of all ___ in the domain of ___ such that _____ is in the domain of ___.

Ex 1.
Let $f(x) = x^2$ and $g(x) = x - 3$. Find the functions $f \circ g$ and $g \circ f$. Also find their domains.

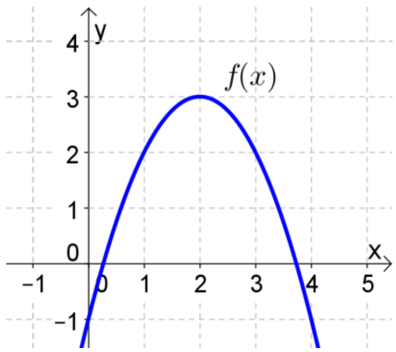
$(f \circ g)(5) =$

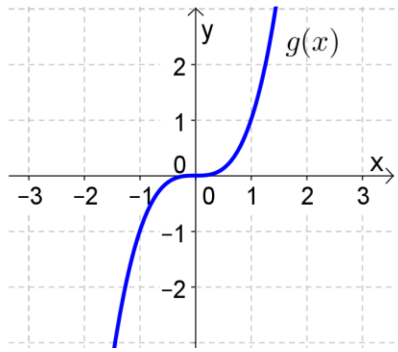
$(g \circ f)(7) =$

Ex 2.
Let $f(x) = \sqrt{x}$ and $g(x) = \sqrt{2 - x}$. Find the functions $f \circ g$ and $g \circ f$. Also find their domains.

A function is ____________ if $f(x_1) \neq f(x_2)$ whenever $x_1 \neq x_2$.
In other words, a function is one-to-one if two different inputs always give you two different outputs.

We can test this visually by using the ________________________________, which says that a function is one-to-one if no horizontal line intersects its graph more than once.





Inverse Functions

Let f be a one-to-one function with domain A and range B.
The _____________ of f, called f^{-1}, has domain B and range A, and is defined by
$f^{-1}(y) = x \quad \Leftrightarrow \quad f(x) = y$ for any y in B. So, any input/output pair of f is switched for f^{-1}.

Inverse functions "undo" other functions. For example, if $f(x) = 2x + 1$, then $f^{-1}(x) = \frac{x-1}{2}$.

To show that $2x + 1$ and $\frac{x-1}{2}$ are inverse functions, we can use the following properties:
$f^{-1}(f(x)) =$ ___ for every x in A
$f(f^{-1}(x)) =$ ___ for every x in B

Ex 3.

Show that $f(x) = 2x + 1$ and $g(x) = \frac{x-1}{2}$ are inverses of each other.

Note: Only one-to-one functions have inverses. (Also, one-to-one functions always have inverses.)

How to find the inverse of $f(x)$:

1. Replace $f(x)$ with ___.
2. __________ x and y.
3. Solve for ____.
4. Replace y with _________.

Ex 4.

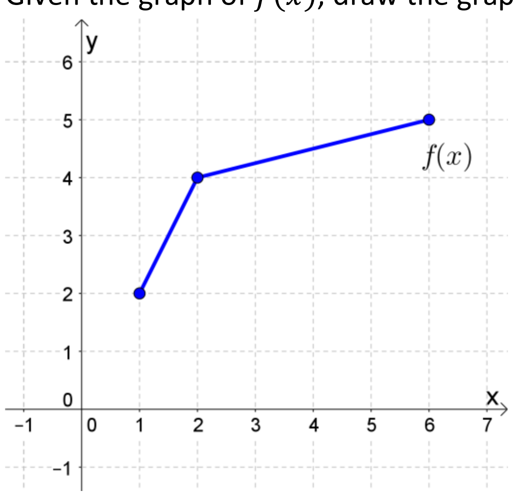
Find the inverse of $f(x) = \frac{2x+3}{x-1}$.

Graphs and Inverses

Given any point (a, b) on the graph of $f(x)$, we can get a point on the graph of $f^{-1}(x)$ by switching the coordinates: (b, a).

Ex 5.

Given the graph of $f(x)$, draw the graph of $f^{-1}(x)$.



Notice that the entire graph of $f^{-1}(x)$ will be the mirror image of $f(x)$ across the $y = x$ line.