1. Let $f(x) = \frac{1}{x}$ and $g(x) = \sqrt{x-2}$.

   a) Find $(g \circ f)\left(\frac{1}{2}\right)$.

$$(g\circ f)\left(\tfrac{1}{2}\right) = g\Big(\underbrace{f\left(\tfrac{1}{2}\right)}_{\frac{1}{(\frac{1}{2})}}\Big) = g(2) = \sqrt{2-2} = \sqrt{0} = \boxed{0}$$

   b) Find $f \circ g$ and its domain.

$$(f\circ g)(x) = f(g(x)) = f\left(\sqrt{x-2}\right) = \boxed{\frac{1}{\sqrt{x-2}}}$$

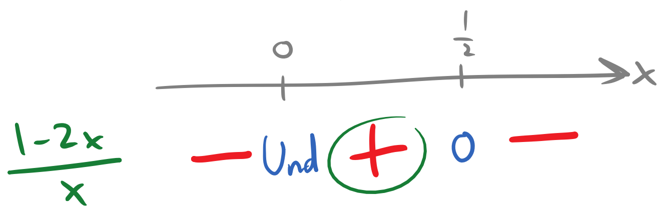
$\boxed{\text{Domain: } (2, \infty)}$

   c) Find $g \circ f$ and its domain.

$$(g\circ f)(x) = g(f(x)) = g\left(\tfrac{1}{x}\right) = \boxed{\sqrt{\tfrac{1}{x} - 2}}$$

Domain: Need $\frac{1}{x} - 2 \geq 0$

$$\frac{1-2x}{x} \geq 0$$



$\boxed{\left(0, \tfrac{1}{2}\right]}$

2. Find the inverse of $f(x) = 2 + \sqrt{x-3}$. Be sure to state the domain of $f^{-1}(x)$. (Hint: the domain of f^{-1} is the range of f)

$$y = 2 + \sqrt{x-3}$$

$$x = 2 + \sqrt{y-3}$$

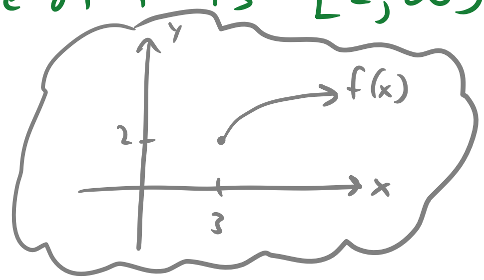
$$x - 2 = \sqrt{y-3}$$

$$(x-2)^2 = y - 3$$

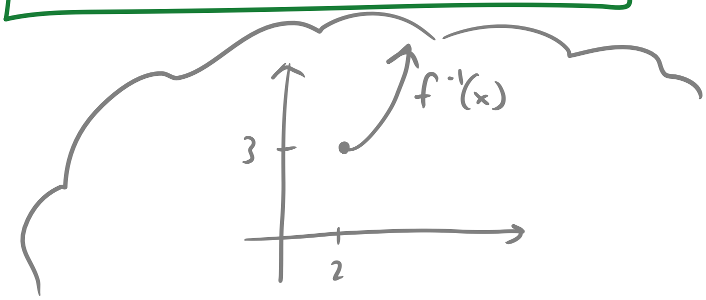
$$y = (x-2)^2 + 3$$

$$\boxed{f^{-1}(x) = (x-2)^2 + 3}$$

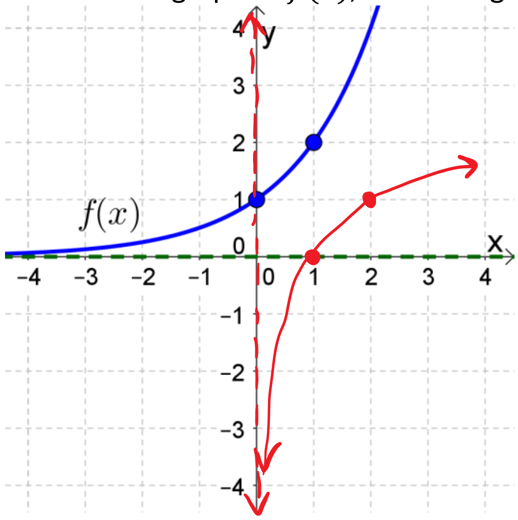
Range of f is $[2, \infty)$



So, $\boxed{\text{domain of } f^{-1} \text{ is } [2, \infty).}$



3. Given the graph of $f(x)$, draw the graph of $f^{-1}(x)$.



Q: What word starts with "e" and has only one letter in it?

4. Let $f(x) = \frac{x}{x-1}$ and $g(x) = x^2 - 3$.

a) Find $(f \circ g)(0)$.

$$f(g(0)) = f(0^2-3) = f(-3) = \frac{-3}{-3-1} = \boxed{\frac{3}{4}}$$

b) Find $(f \circ g)(2)$.

$$f(g(2)) = f(2^2-3) = f(1) = \frac{1}{1-1} = \frac{1}{0} \quad \boxed{\text{undefined}}$$

c) Find $(g \circ g)(3)$.

$$g(g(3)) = g(3^2-3) = g(6) = 6^2 - 3 = \boxed{33}$$

d) Find $f \circ g$ and its domain.

$$f(g(x)) = f(x^2-3) = \frac{x^2-3}{x^2-3-1} = \boxed{\frac{x^2-3}{x^2-4}}$$

$$\boxed{\text{Domain: } \{x \mid x \neq \pm 2\}} \leftarrow \begin{array}{l} x^2-4 \neq 0 \\ x^2 \neq 4 \\ x \neq \pm 2 \end{array}$$

e) Find $g \circ f$ and its domain.

$$g(f(x)) = g\left(\frac{x}{x-1}\right) = \boxed{\left(\frac{x}{x-1}\right)^2 - 3}$$

$$\boxed{\text{Domain: } \{x \mid x \neq 1\}}$$

f) Find $f \circ f$ and simplify it. Based on what you got for $f \circ f$, what can you say about the inverse of f?

$$f(f(x)) = f\left(\frac{x}{x-1}\right)$$

$$= \frac{\left(\frac{x}{x-1}\right)\cdot\cancel{(x-1)}}{\left(\frac{x}{x-1}-1\right)\cdot(x-1)}$$

$$= \frac{x}{x-(x-1)}$$

$$= \frac{x}{1}$$

$$= \boxed{x}$$

The inverse of f is $f^{-1}(x) = \frac{x}{x-1}$.

(This is because we got $f(f(x)) = x$, so f is its own inverse!)

6. Show that $f(x) = \frac{1}{x-1}$ and $g(x) = \frac{1}{x} + 1$ are inverses of each other.

$$f(g(x)) = f\left(\frac{1}{x}+1\right) = \frac{1}{\frac{1}{x}+1-1} = \frac{1}{(1/x)} = x \quad \checkmark$$

$$g(f(x)) = g\left(\frac{1}{x-1}\right) = \frac{1}{\left(\frac{1}{x-1}\right)} + 1 = x-1+1 = x \quad \checkmark$$

8. Find the inverse of $f(x) = \frac{x-1}{2x+3}$. Be sure to state the domain of $f^{-1}(x)$.

$$y = \frac{x-1}{2x+3}$$

Switch x and y

$$x = \frac{y-1}{2y+3}$$

$$2xy + 3x = y - 1$$

$$2xy - y = -3x - 1$$

$$y(2x-1) = -3x-1$$

$$y = \frac{-3x-1}{2x-1}$$

$$\boxed{f^{-1}(x) = \frac{-3x-1}{2x-1}}$$

$$\boxed{\text{Domain: } \left\{x \mid x \neq \tfrac{1}{2}\right\}}$$

10. Given the graph of $f(x)$, draw the graph of $f^{-1}(x)$.