1. Suppose $f(x) = x^2 + 1$. Find $\frac{f(x+h)-f(x)}{h}$ and simplify by canceling the factor of h.

$f(x+h) = (x+h)^2 + 1$

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2+1-(x^2+1)}{h}$$

$$= \frac{\cancel{x^2}+2xh+h^2\cancel{+1}-\cancel{x^2}-\cancel{1}}{h}$$

$$= \frac{2xh+h^2}{h}$$

$$= \boxed{2x+h}$$

2. Suppose $f(x) = \begin{cases} -5 & \text{if } x < -1 \\ x^2+1 & \text{if} -1 \leq x < 2 \\ 2-x & \text{if } x \geq 2 \end{cases}$

a) Find $f(1)$.

$f(1) = 1^2 + 1 = \boxed{2}$

b) Find $f(2)$.

$f(2) = 2 - 2 = \boxed{0}$

c) Find $f(-2)$.

$f(-2) = \boxed{-5}$

d) Find the domain of f.

$\boxed{(-\infty, \infty)}$ (OR $\boxed{\mathbb{R}}$)

3. Find the domain of $f(x) = \frac{x^4}{x^2+x-6}$.

Need $x^2 + x - 6 \neq 0$

$(x+3)(x-2) \neq 0$

$x \neq -3$ and $x \neq 2$

Domain: $\boxed{\{x \mid x \neq -3 \text{ and } x \neq 2\}}$

(OR $(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$)

4. Suppose $f(x) = \frac{\sqrt{3-x}}{x^2-2x}$.

a) Find $f(-1), f(0), f(1), f(2), f(3), f(4)$.

$f(-1) = \frac{\sqrt{3-(-1)}}{(-1)^2-2(-1)} = \frac{\sqrt{4}}{1+2} = \boxed{\frac{2}{3}}$

$f(0) = \frac{\sqrt{3-0}}{0^2-2(0)} = \frac{\sqrt{3}}{0}$ $\boxed{\text{undefined}}$

$f(1) = \frac{\sqrt{3-1}}{1^2-2(1)} = \frac{\sqrt{2}}{1-2} = \boxed{-\sqrt{2}}$

$f(2) = \frac{\sqrt{3-2}}{2^2-2(2)} = \frac{\sqrt{1}}{4-4} = \frac{1}{0}$ $\boxed{\text{undefined}}$

$f(3) = \frac{\sqrt{3-3}}{3^2-2(3)} = \frac{\sqrt{0}}{9-6} = \frac{0}{3} = \boxed{0}$

$f(4) = \frac{\sqrt{3-4}}{4^2-2(4)} = \frac{\sqrt{-1}}{16-8}$ $\boxed{\text{undefined}}$

b) Find the domain of f.

Need $3-x \geq 0$ and $x^2-2x \neq 0$

$3 \geq x$ and $x(x-2) \neq 0$

$x \leq 3$ and $x \neq 0$ and $x \neq 2$

Domain: $\boxed{\{x \mid x \leq 3 \text{ and } x \neq 0 \text{ and } x \neq 2\}}$

(OR $(-\infty, 0) \cup (0, 2) \cup (2, 3]$)

6. Suppose $f(x) = x^2 + x$.

a) Find $\frac{f(x+h)-f(x)}{h}$ and simplify by canceling the factor of h.

$f(x+h) = (x+h)^2 + (x+h)$

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2+(x+h)-(x^2+x)}{h}$$

$$= \frac{\cancel{x^2}+2xh+h^2+\cancel{x}+h-\cancel{x^2}-\cancel{x}}{h}$$

$$= \frac{2xh+h^2+h}{h} = \boxed{2x+h+1}$$

b) Find the domain of f.

$\boxed{(-\infty, \infty)}$ $\left(\text{or } \mathbb{R}\right)$

all real numbers

8. Suppose $f(x) = \frac{2}{x+1}$.

a) Find $\frac{f(x+h)-f(x)}{h}$ and simplify by canceling the factor of h.

$f(x+h) = \frac{2}{x+h+1}$

LCD: $(x+h+1)(x+1)$

$$\frac{f(x+h)-f(x)}{h} = \frac{\left(\frac{2}{x+h+1} - \frac{2}{x+1}\right)\cdot(x+h+1)(x+1)}{(h)\cdot(x+h+1)(x+1)}$$

$$= \frac{2(x+1) - 2(x+h+1)}{h(x+h+1)(x+1)}$$

$$= \frac{\cancel{2x} \cancel{+2} \cancel{-2x} - 2h \cancel{-2}}{h(x+h+1)(x+1)}$$

$$= \frac{-2\cancel{h}}{\cancel{h}(x+h+1)(x+1)} = \boxed{\frac{-2}{(x+h+1)(x+1)}}$$

b) Find the domain of f.

$\boxed{\{x \mid x \neq -1\}}$ $\left(\text{OR } (-\infty, -1) \cup (-1, \infty)\right)$

10. Suppose $f(x) = \begin{cases} \frac{1}{x} & \text{if } x < -1 \\ \sqrt[3]{x} & \text{if } x > -1 \end{cases}$

a) Find $f(-2)$, $f(-1)$, $f\left(-\frac{1}{8}\right)$, $f(0)$, $f(27)$.

$f(-2) = \frac{1}{-2} = \boxed{-\frac{1}{2}}$

$f(-1)$ $\boxed{\text{undefined}}$

$f\left(-\frac{1}{8}\right) = \sqrt[3]{-\frac{1}{8}} = \boxed{-\frac{1}{2}}$

$f(0) = \sqrt[3]{0} = \boxed{0}$

$f(27) = \sqrt[3]{27} = \boxed{3}$

b) Find the domain of f.

$\boxed{\{x \mid x \neq -1\}}$ $\left(\text{OR } (-\infty, -1) \cup (-1, \infty)\right)$