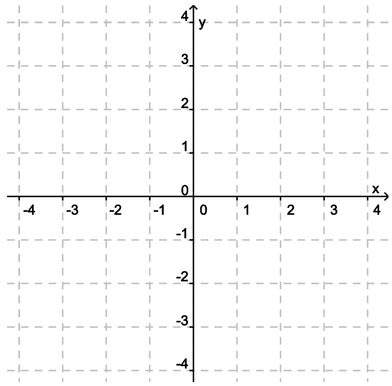
Graphs of Functions

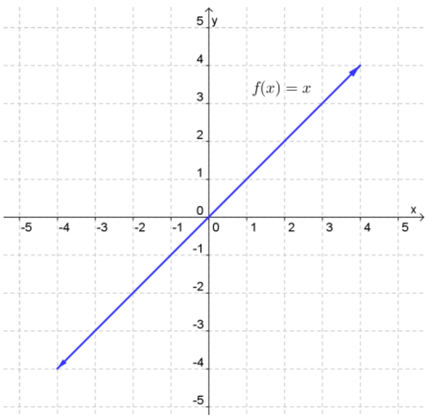
(covers parts of Sullivan 2.3, 2.4, and 2.5)

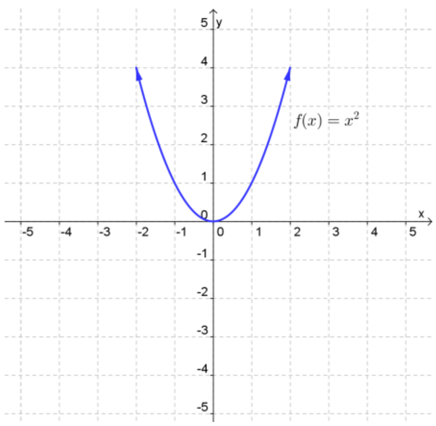
Ex 1.

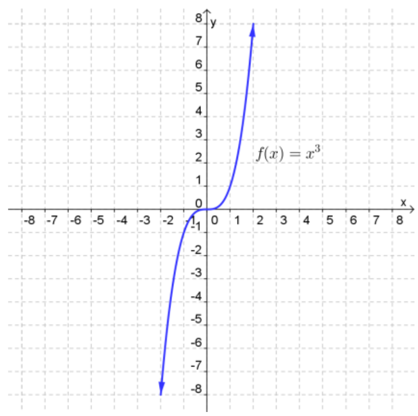
Graph $f(x) = \sqrt{x}$.

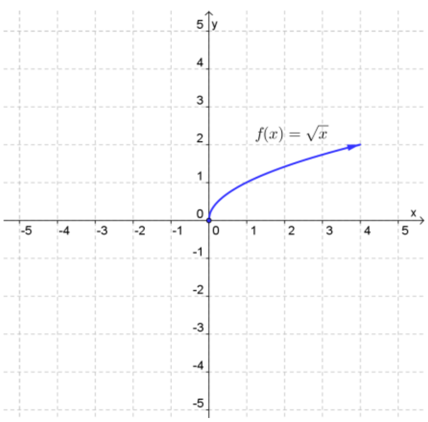


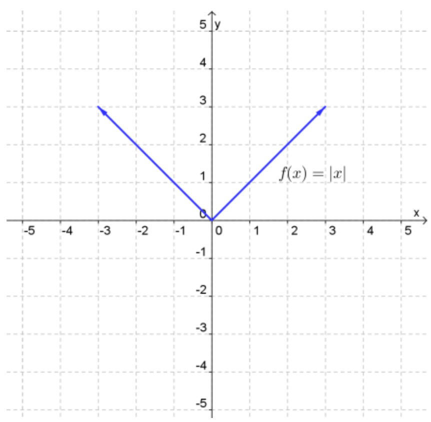
Graphs of Basic Functions

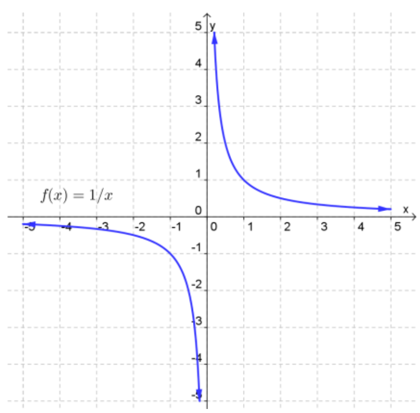




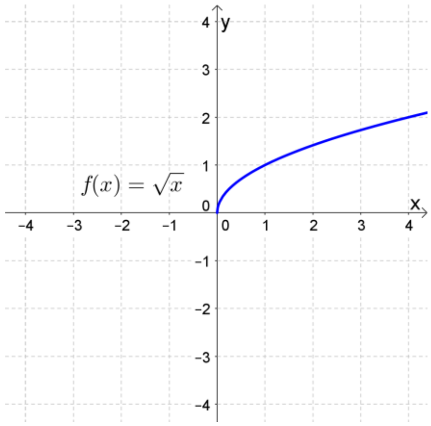


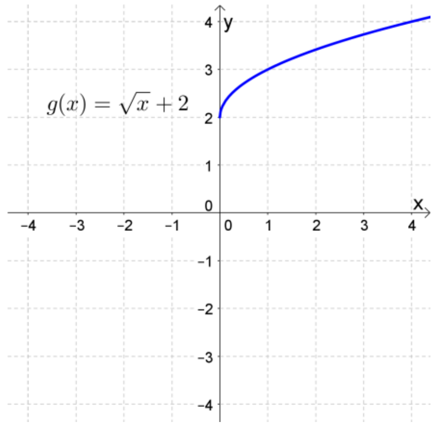






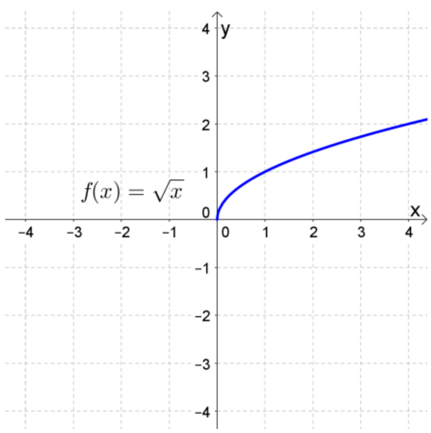
What happens to the graph of $f(x) = \sqrt{x}$ if we add 2 to all the outputs/y-values?
In other words, what does $g(x) = \sqrt{x} + 2$ look like?

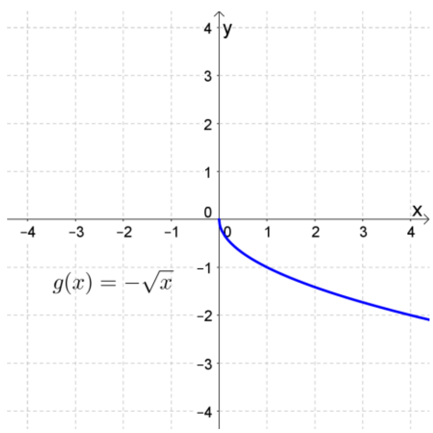




x	$\sqrt{x}$	$\sqrt{x} + 2$
0	0	$0 + 2 = 2$
1	1	$1 + 2 = 3$
4	2	$2 + 2 = 4$

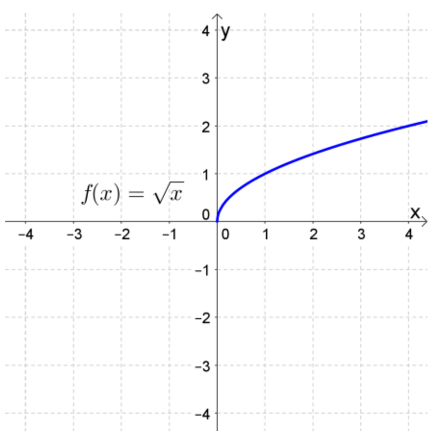
What if we multiplied the outputs of $f(x) = \sqrt{x}$ by negative 1?
In other words, what does $g(x) = -\sqrt{x}$ look like?

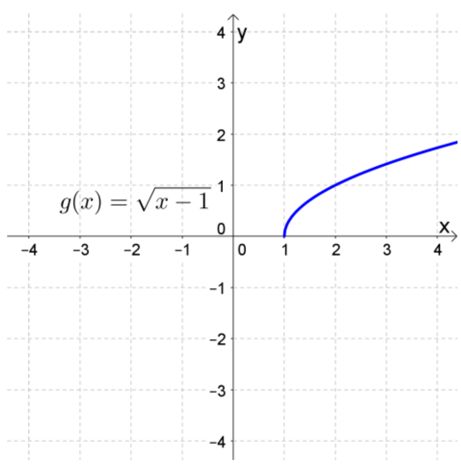




x	$\sqrt{x}$	$-\sqrt{x}$
0	0	$-0 = 0$
1	1	-1
4	2	-2

Now, what happens if we replace x with $x - 1$? So, $g(x) = \sqrt{x - 1}$.
Notice that plugging in $x = 1$ into $\sqrt{x - 1}$ is like plugging in $x = 0$ into $\sqrt{x}$.
And plugging in $x = 5$ into $\sqrt{x - 1}$ is like plugging in $x = 4$ into $\sqrt{x}$.
So, the effect of $\underline{x - 1}$ is to "pull" the y-values of $\sqrt{x}$ to the right by 1.





x	$\sqrt{x - 1}$
1	$\sqrt{1 - 1} = \sqrt{0} = 0$
2	$\sqrt{2 - 1} = \sqrt{1} = 1$
5	$\sqrt{5 - 1} = \sqrt{4} = 2$

Transformations of Functions

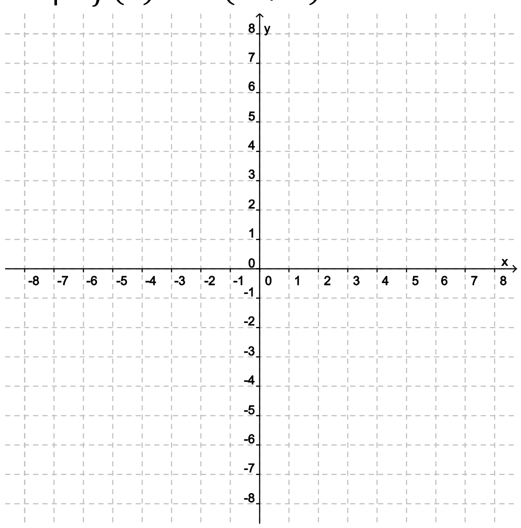
For a function $f(x)$, and $c > 0$,
$f(x) + c$ shifts up
$f(x) - c$ shifts down
$cf(x)$ stretches vertically (if $c > 1$), or shrinks vertically (if $0 < c < 1$)
$-f(x)$ reflects about x-axis

$f(x - c)$ shifts right
$f(x + c)$ shifts left
$f(cx)$ stretches horizontally (if $0 < c < 1$), or shrink horizontally (if $c > 1$)
$f(-x)$ reflects about y-axis

Note: To graph a transformed function, plot key points of basic function and then move points based on transformations.

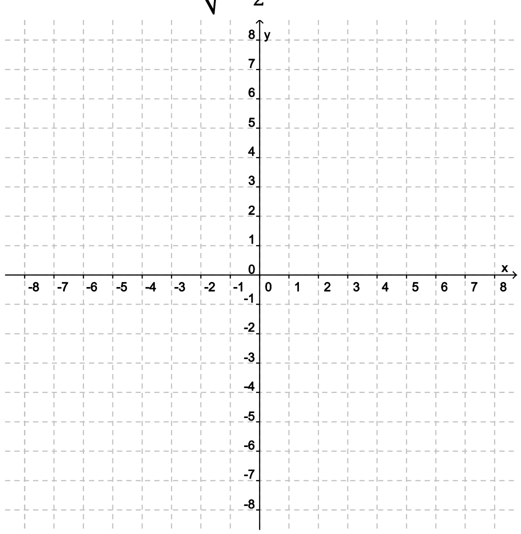
Ex 2.

Graph $f(x) = -(x + 4)^2$. Be sure to describe the transformations to the basic function.



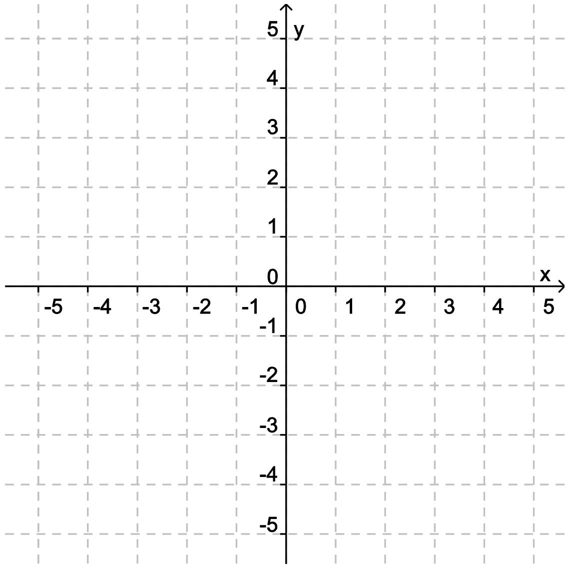
Ex 3.

Graph $g(x) = \sqrt{-\frac{1}{2}x + 3}$. Be sure to describe the transformations to the basic function.



Ex 4.

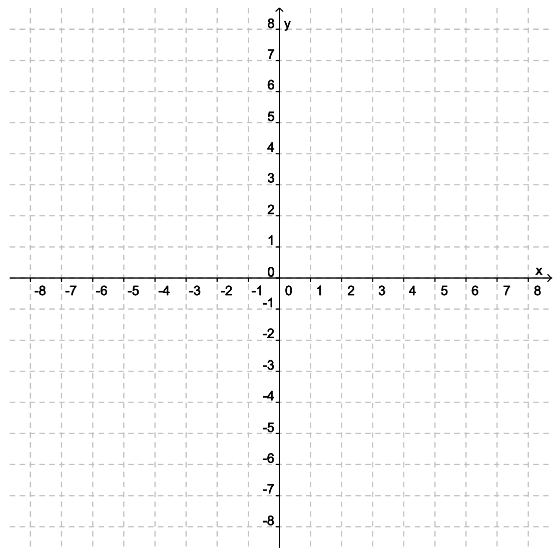
Graph $h(x) = \frac{1}{2}|x - 1| - 2$. Be sure to describe the transformations to the basic function.



Piecewise-Defined Function

Ex 5.

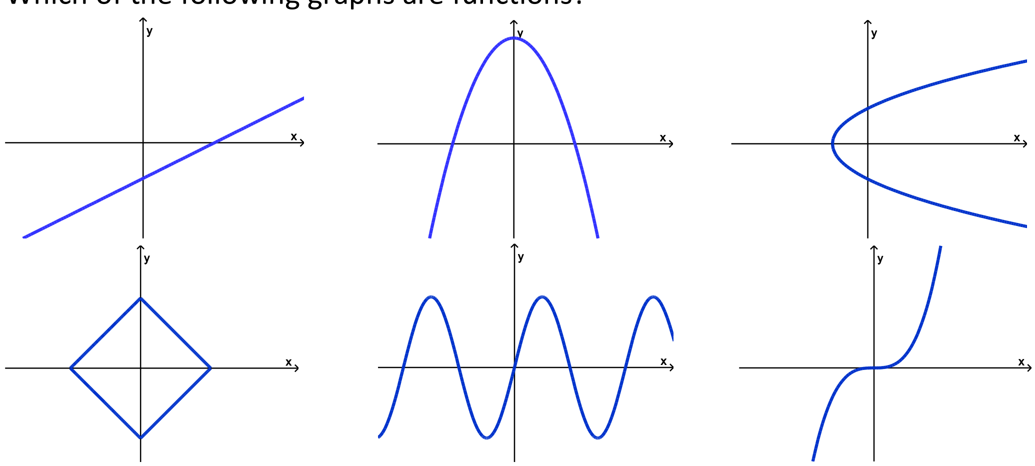
Graph $f(x) = \begin{cases} 2x + 7 & \text{if } x \leq -3 \\ 4 & \text{if} -3 < x \leq 0. \\ x^2 - 2 & \text{if } x > 0 \end{cases}$



To test if a graph is a function we can use the ____________________________________. If any vertical line intersects a graph in two or more points, then the graph does not represent a function.

Ex 6.

Which of the following graphs are functions?



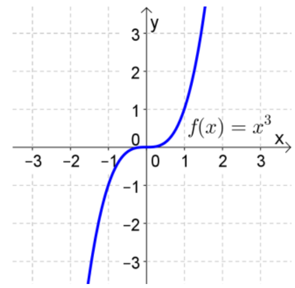
Even and Odd Functions

f is an ____________________ if $f(-x) = f(x)$ for all x in the domain of f.

f is an ____________________ if $f(-x) = -f(x)$ for all x in the domain of f.

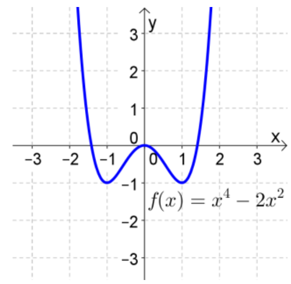
Ex 7.

Determine whether $f(x) = x^3$ is even, odd, or neither.



Ex 8.

Determine whether $f(x) = x^4 - 2x^2$ is even, odd, or neither.



Note:

Even functions are symmetric about the _________.

Odd functions are symmetric about the _________.