_____ / 15 total points

Quiz #3 Name: Solutions^{-1} x

Math 150, Prof. Beydler Thursday, May 4, 2017

Directions: Show all work. No books or notes. A **scientific calculator** is allowed. Your desk and lap must be clear (no phones, notebooks, etc.). Write your answers in the indicated places, or box your answers. Good luck!

1. (3 points) Verify that the following equation is an identity.

$\tan x + \cot x = 2\csc 2x$

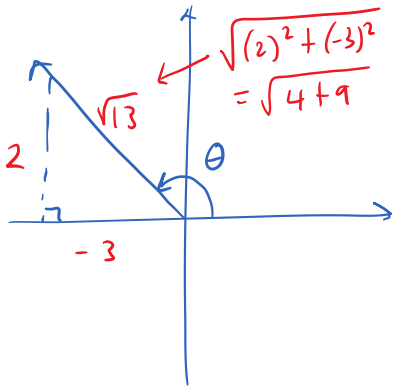
$RHS = 2\csc 2x = \frac{2}{\sin 2x} = \frac{2}{2\sin x\cos x} = \frac{1}{\sin x\cos x}$

$LHS = \tan x + \cot x = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\cos x \sin x} = \frac{1}{\cos x \sin x}$

$LHS = \frac{1}{\sin x\cos x} = RHS$ ▨

2. (3 points) Find the exact value of $\cos 2\theta$ given that $\tan\theta = -\frac{2}{3}$ and $\sin\theta > 0$.

Answer: $\frac{5}{13}$



$\cos 2\theta = 2\cos^2\theta - 1$

$= 2\left(\frac{-3}{\sqrt{13}}\right)^2 - 1$

$= 2\left(\frac{9}{13}\right) - 1$

$= \frac{18}{13} - 1$

3. (3 points) Find the exact value of $\sin 112.5°$. Be sure to show your work here. You can check your answer on the calculator, but you need to show how to get that answer by hand.

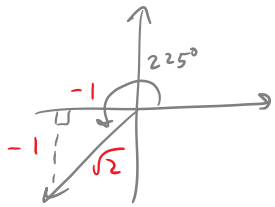
Answer: $\frac{\sqrt{2+\sqrt{2}}}{2}$

$\sin 112.5° = \sin\frac{225°}{2}$

$= \pm\sqrt{\frac{1-\cos 225°}{2}}$

$= +\sqrt{\frac{\left(1-\left(-\frac{\sqrt{2}}{2}\right)\right)\cdot 2}{(2)\cdot 2}}$

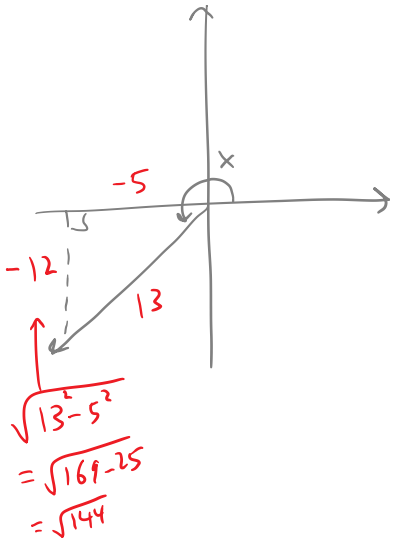
$= \sqrt{\frac{2+\sqrt{2}}{4}}$



$\cos 225° = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

4. (3 points) Given $\cos x = -\frac{5}{13}$ and $\pi < x < \frac{3\pi}{2}$, find the exact value of $\cos\frac{x}{2}$.

Answer: $-\frac{2}{\sqrt{13}}$



$\sqrt{13^2-5^2}$
$=\sqrt{169-25}$
$=\sqrt{144}$

$$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$$

$$= -\sqrt{\frac{\left(1+\left(\frac{-5}{13}\right)\right)\cdot 13}{(2)\cdot 13}}$$

$$= -\sqrt{\frac{13-5}{26}}$$

$$= -\sqrt{\frac{8}{26}}$$

$\pi < x < \frac{3\pi}{2}$

$\frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$

$\frac{x}{2}$ in QII, so $\cos\frac{x}{2} < 0$

5. (3 points) Write $\sin(2\underbrace{\cos^{-1}x}_{\theta})$ as an algebraic expression in x.

Answer: $2x\sqrt{1-x^2}$

$= \sin 2\theta$

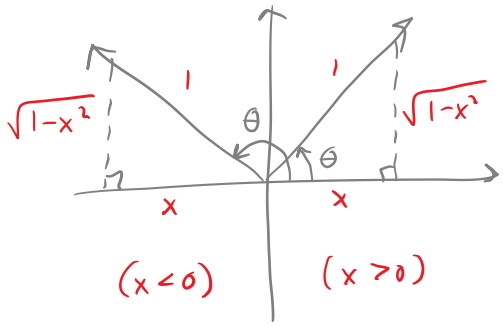
$= 2\sin\theta\cos\theta$

$= 2\left(\frac{\sqrt{1-x^2}}{1}\right)\left(\frac{x}{1}\right)$

$= 2x\sqrt{1-x^2}$

$\cos^{-1}x = \theta$ $\leftarrow [0, \pi]$

$\cos\theta = x = \frac{x}{1}$



In case you need them (though you probably won't), here are the formulas for tangent that I promised:

$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$ $\quad \tan\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{1+\cos A}}$ $\quad \tan\frac{A}{2} = \frac{\sin A}{1+\cos A}$ $\quad \tan\frac{A}{2} = \frac{1-\cos A}{\sin A}$