Math 150 – Final Exam Study Guide

Spring 2017, Prof. Beydler

Test Info

- Date: Thursday, June 15, 2017 from 7:30am to 10:00am
- Will cover almost all sections.
- There will be 2 parts to the test:
  - Part 1: No calculators allowed.
  - Part 2: **Scientific calculator** allowed (you'll need it!).
- When you're finished with Part 1, turn it in and I'll give you Part 2.
- No notes, no books, no phones during the final exam. Please don't fail the class because of a phone in your lap!
- As usual, there will be a seating chart for the final exam.
- Where to get help as you're studying:
  - Office hours
  - TMARC, LAC, or other tutoring centers
  - E-mail me at dbeydler@mtsac.edu
- If you go to the TMARC/LAC for 4 hours between Test #3 and the Final Exam, you'll get 1% extra credit towards the Final Exam.

Not on the final exam:

- Cofunction identities from 2.1.
- The ambiguous SSA case using the Law of Sines. (7.2)

Formulas and stuff

(Note: Know all of these except for the ones with  next to them, which I'll give you. This list is not meant to include everything you'll need to know on the test.)

- Complementary angles add up to $90°$. Supplementary angles add up to $180°$.
- $1° = 60', \ 1' = 60''$

Trig function definitions ($r = \sqrt{x^2 + y^2}$):

$\sin\theta = \frac{y}{r}$ $\qquad \csc\theta = \frac{r}{y}$

$\cos\theta = \frac{x}{r}$ $\qquad \sec\theta = \frac{r}{x}$

$\tan\theta = \frac{y}{x}$ $\qquad \cot\theta = \frac{x}{y}$

Reciprocal Identities

$\sin\theta = \frac{1}{\csc\theta}$ $\qquad \csc\theta = \frac{1}{\sin\theta}$

$\cos\theta = \frac{1}{\sec\theta}$ $\qquad \sec\theta = \frac{1}{\cos\theta}$

$\tan\theta = \frac{1}{\cot\theta}$ $\qquad \cot\theta = \frac{1}{\tan\theta}$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

Quotient Identities

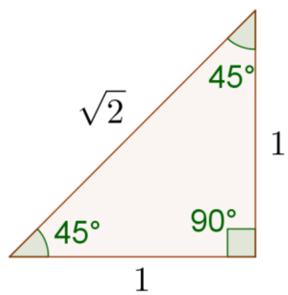
$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$

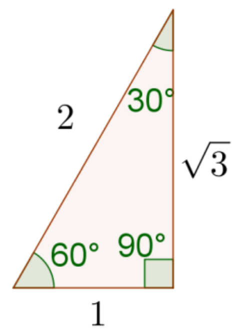
Trig Function of θ	Range
$\sin\theta,\ \cos\theta$	$[-1,1]$
$\tan\theta,\ \cot\theta$	$(-\infty,\infty)$
$\sec\theta,\ \csc\theta$	$(-\infty,-1]\cup[1,\infty)$

- **Trig functions in right triangles**

  $\sin\theta=\frac{\text{opp}}{\text{hyp}}$ $\quad\cos\theta=\frac{\text{adj}}{\text{hyp}}$ $\quad\tan\theta=\frac{\text{opp}}{\text{adj}}$

  $\csc\theta=\frac{\text{hyp}}{\text{opp}}$ $\quad\sec\theta=\frac{\text{hyp}}{\text{adj}}$ $\quad\cot\theta=\frac{\text{adj}}{\text{opp}}$



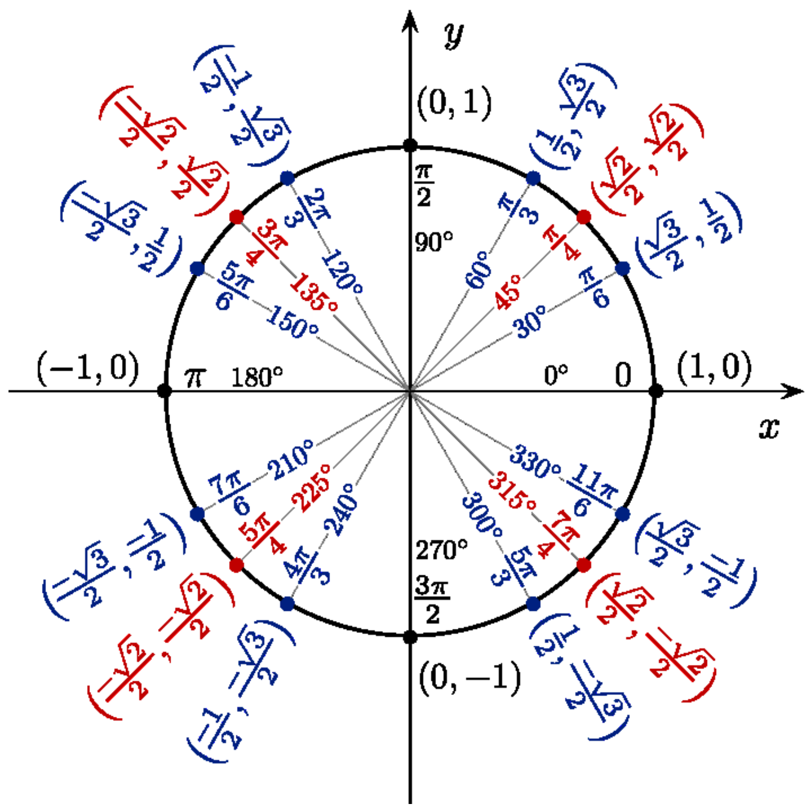


- **Radian measure of θ:** $\theta=\frac{s}{r}$
- $180^\circ=\pi$ radians
- **Arc length:** $s=\theta r$
- **Area of a sector:** $\frac{1}{2}r^2\theta$
- **Trig functions on the unit circle:**

  $\sin s=y$ $\qquad\csc s=\frac{1}{y}$

  $\cos s=x$ $\qquad\sec s=\frac{1}{x}$

  $\tan s=\frac{y}{x}$ $\qquad\cot s=\frac{x}{y}$



- **Linear speed:** $v=\frac{s}{t}$
- **Angular speed:** $\omega=\frac{\theta}{t}$
- $v=\omega r$
- Period of $\sin x$, $\cos x$, $\csc x$, $\sec x$: 2π $(0\le x\le 2\pi)$
- Period of $\tan x$: π $\left(-\frac{\pi}{2}<x<\frac{\pi}{2}\right)$
- Period of $\cot x$: π $(0<x<\pi)$

Reciprocal Identities

$\sin\theta = \frac{1}{\csc\theta}$ $\csc\theta = \frac{1}{\sin\theta}$

$\cos\theta = \frac{1}{\sec\theta}$ $\sec\theta = \frac{1}{\cos\theta}$

$\tan\theta = \frac{1}{\cot\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

Quotient Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Sum/Difference Identities

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \frac{\tan A+\tan B}{1-\tan A\tan B}$

$\tan(A-B) = \frac{\tan A-\tan B}{1+\tan A\tan B}$

Double-Angle Identities

$\cos 2A = \cos^2 A - \sin^2 A$ $\cos 2A = 1 - 2\sin^2 A$ $\cos 2A = 2\cos^2 A - 1$

$\sin 2A = 2\sin A\cos A$

$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

Half-Angle Identities

$\sin\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}}$

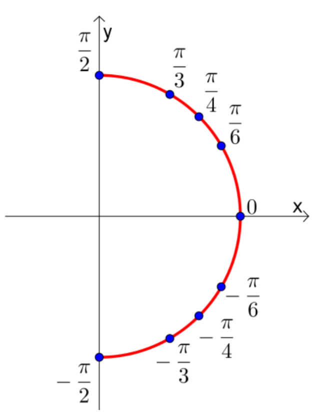
$\cos\frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{2}}$

$\tan\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{1+\cos A}}$ $\tan\frac{A}{2} = \frac{\sin A}{1+\cos A}$ $\tan\frac{A}{2} = \frac{1-\cos A}{\sin A}$

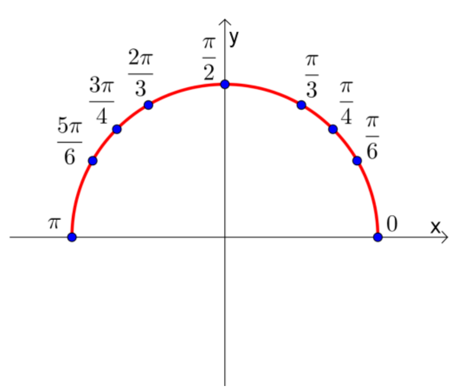
- Inputs and outputs of inverse functions:

	Domain	Range
$\sin^{-1} x$	$[-1,1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1} x$	$[-1,1]$	$[0,\pi]$
$\tan^{-1} x$	$(-\infty,\infty)$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\sin^{-1} x$ and $\tan^{-1} x$ output:



$\cos^{-1} x$ outputs:



- The Law of Sines: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ (also written $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$)
- The Law of Cosines: $a^2 = b^2 + c^2 - 2bc\cos A$
- Area of a triangle: $Area = \frac{1}{2}bc\sin A$
- Vectors
  - Magnitude of $\vec{v} = \langle a, b \rangle$ is $|\vec{v}| = \sqrt{a^2 + b^2}$
  - Direction angle satisfies $\tan\theta = \frac{b}{a}$
  - Vectors with length 1 are called unit vectors.
  - $\vec{\imath} = \langle 1,0 \rangle$ and $\vec{\jmath} = \langle 0,1 \rangle$
  - $\vec{v} = \langle a, b \rangle = a\vec{\imath} + b\vec{\jmath}$
  - Horizontal component: $|\vec{v}|\cos\theta$, Vertical component: $|\vec{v}|\sin\theta$
  - The dot product of $\vec{u} = \langle a, b \rangle$ and $\vec{v} = \langle c, d \rangle$ is: $\vec{u} \cdot \vec{v} = ac + bd$
  - The Dot Product Theorem: $\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}|\cos\theta$ (θ is the angle between $\vec{u}$ and $\vec{v}$)
  - Two nonzero vectors $\vec{u}$ and $\vec{v}$ are perpendicular (orthogonal) if and only if $\vec{u} \cdot \vec{v} = 0$.
  - The equilibrant of a resultant vector is the negative of that vector.
- Complex numbers
  - $i^2 = -1$
  - Rectangular and trigonometric form
    - $x = r\cos\theta$, $y = r\sin\theta$
    - $r = \sqrt{x^2 + y^2}$, $\tan\theta = \frac{y}{x}$
  - Product Theorem:
    $[r_1(\cos\theta_1 + i\sin\theta_1)] \cdot [r_2(\cos\theta_2 + i\sin\theta_2)] = r_1 r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$
  - Quotient Theorem:
    $\frac{r_1(\cos\theta_1 + i\sin\theta_1)}{r_2(\cos\theta_2 + i\sin\theta_2)} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$
  - De Moivre's Theorem:
    $[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$
  - nth Root Theorem:
    The nth roots of $r(\cos\theta + i\sin\theta)$ are given by $\sqrt[n]{r}(\cos\alpha + i\sin\alpha)$, where $\alpha = \frac{\theta}{n} + \frac{360°}{n} \cdot k$ or $\alpha = \frac{\theta}{n} + \frac{2\pi}{n} \cdot k$ for $k = 0,1,2, \ldots, n-1$.
- Polar coordinates
  - Rectangular and polar form
    - $x = r\cos\theta$, $y = r\sin\theta$
    - $r^2 = x^2 + y^2$, $\tan\theta = \frac{y}{x}$

Extra Credit!

- If you write up the answers to all of the review exercises (see Final Exam Review Exercises on the class website), and hand them in at the test, you can earn up to 3% extra credit towards your test (depending on neatness and completeness)! Note that these review exercises don't necessarily cover everything.
- If you go to the TMARC/LAC for 4 hours between Test #3 and the Final Exam, you'll get 1% extra credit towards the Final Exam.