Math 150 – Final Exam Review Exercises

Note: It is important to understand that these review exercises are not guaranteed to cover all of the potential problems on the final exam. Please review the notes, practice problems, tests, and homework problems to fully prepare for the final exam. Now, take a deep breath, and get to it! ☺

1. Suppose that angle θ is in quadrant II and $\cos\theta = -\frac{2}{5}$. Find the exact values of the other 5 trig functions.

2. Decide whether each statement is *possible* or *impossible*.
   a. $\sin\theta = -\frac{3}{2}$
   b. $\sec\theta = -\frac{3}{2}$
   c. $\tan\theta = 1000000$
   d. $\sin^{-1} x = \frac{2\pi}{3}$
   e. $\cos^{-1} x = \frac{2\pi}{3}$

3. Evaluate the following without a calculator. (Write your answers as exact values.)
   a. $\sin(225°)$
   b. $\cos(-180°)$
   c. $\tan(1560°)$
   d. $\csc(-450°)$
   e. $\sec(675°)$
   f. $\cot(7200°)$
   g. $\sin\left(\frac{7\pi}{3}\right)$
   h. $\cos\left(-\frac{9\pi}{4}\right)$
   i. $\tan\left(\frac{13\pi}{4}\right)$
   j. $\csc\left(\frac{5\pi}{3}\right)$
   k. $\sec\left(\frac{11\pi}{6}\right)$
   l. $\cot\left(-\frac{5\pi}{6}\right)$
   m. $\cos 67.5°$
   n. $\sin 195°$
   o. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$
   p. $\cot^{-1}(-1)$
   q. $\sec^{-1}(-2)$
   r. $\sin^{-1} 0$
   s. $\sin^{-1}\left(-\frac{1}{2}\right)$
   t. $\csc^{-1}\sqrt{2}$
   u. $\tan^{-1}(-\sqrt{3})$

4. You're looking at a flagpole, and find that the angle of elevation from a point 24.7 ft from the base of the pole to the top of the pole is $32°10'$. What is the height of the pole?

5. The angle of depression from a tower to a point on the ground 36.0 m from the bottom of the tower is $29.5°$. Find the height of the tower.

6. Convert $-450°$ to radians.

7. Convert $\frac{11\pi}{4}$ to degrees.

8. Two gears (with radii 4.8 in and 7.1 in) are aligned so that the smaller gear drives the larger one. If the smaller gear rotates $315°$, how many degrees will the larger gear rotate?

9. Find the radius of a pulley that raises a weight 11.4 cm with a rotation of $51.6°$.

10. The tires of a bicycle have radius 14 in. and are turning at the rate of 200 revolutions per min. How fast is the bicycle traveling in miles per hour? (Note: 1 mile = 5280 feet.)

11. Find the angular speed ω of the second hand of a clock in radians per minute.

12. Graph $y = 3\cos\left(\frac{2}{3}x - \frac{\pi}{6}\right)$ over a one-period interval and find the amplitude, period, and phase shift. Be sure to show the 5 key x-values.

13. Graph $y = -1 - 4\sin 2x$ over a one-period interval and the amplitude, period, and phase shift. Be sure to show the 5 key x-values.

14. Graph $y = \tan\frac{1}{2}\left(x + \frac{\pi}{4}\right)$ over a one-period interval and find the period. Be sure to show the 5 key x-values.

15. Graph $y = 2\sec(2\pi x - \pi)$ over a one-period interval and find the period. Be sure to show the 5 key x-values.

16. Verify that each equation is an identity.
    a. $\frac{\sin x}{\cos x} + \frac{\cos x}{1+\sin x} = \sec x$
    b. $\tan x - \tan y = \frac{\sin(x-y)}{\cos x \cos y}$
    c. $2\cos^3 x - \cos x = \frac{\cos^2 x - \sin^2 x}{\sec x}$
    d. $\csc x \sin 2x - \sec x = \cos 2x \sec x$
    e. $\frac{1}{1+\cos x} - \frac{1}{1-\cos x} = -2\cot x \csc x$
    f. $\frac{\cos x + 1}{\tan^2 x} = \frac{\cos x}{\sec x - 1}$
    g. $\frac{\sin(x+y)}{\cos(x-y)} = \frac{\cot x + \cot y}{1+\cot x \cot y}$
    h. $\cos 2x = \frac{1-\tan^2 x}{1+\tan^2 x}$

17. Find $\cos(s-t)$ if $\sin s = \frac{2}{3}$ and $\sin t = -\frac{1}{3}$ and s is in Quadrant II and t is in Quadrant IV.

18. Find $\sin(s+t)$ if $\cos s = -\frac{1}{5}$ and $\sin t = \frac{3}{5}$ and s and t are in Quadrant II. Also, which quadrant is $s+t$ in?

19. Find the values of the six trigonometric functions of θ if $\cos 2\theta = -\frac{12}{13}$ and $180° < \theta < 270°$.

20. Find $\sin 2x$ if $\tan x = \frac{5}{3}$ and $\sin x < 0$.

21. Find $\cos\frac{x}{2}$ given that $\sin x = -\frac{4}{5}$ and $\pi < x < \frac{3\pi}{2}$.

22. Find $\sin\frac{x}{2}$, $\cos\frac{x}{2}$, and $\tan\frac{x}{2}$ given that $\tan x = -\frac{\sqrt{5}}{2}$ and $\frac{\pi}{2} < x < \pi$.

23. Find the exact value of each of the following.
    a. $\sin^{-1}\left(\sin\frac{5\pi}{4}\right)$
    b. $\csc\left(\cos^{-1}\frac{7}{25}\right)$

24. Rewrite each of the following as an algebraic expression in x (and possibly y).
    a. $\cos(\tan^{-1} x)$
    b. $\tan(\sin^{-1} x + \cos^{-1} y)$
    c. $\sin(2\cos^{-1} x)$

25. For each trig equation, find all solutions, and then list the solutions in the interval $[0,2\pi)$.
    a. $\sin^2 x \cos x = \cos x$
    b. $\sec\frac{x}{2} = \cos\frac{x}{2}$
    c. $\sin 2x = \cos x$
    d. $\cos^2 x + 5\cos x = -3$
    e. $\cos x + 1 = \sin x$
    f. $2\sin 3x = 1$
    g. $\tan x \sin^2 x = \frac{3}{4}\tan x$

26. Solve each triangle ABC with the given information.
    a. $a = 94.6$ yd, $b = 123$ yd, $c = 109$ yd
    b. $A = 35°$, $B = 15°$, $c = 5$ m
    c. $A = 35.3°$, $B = 52.8°$, $b = 675$ ft
    d. $b = 1$, $c = 3$, $A = 80°$

27. Suppose $\vec{u} = \langle 2, -1\rangle$ and $\vec{v} = \langle 3, 2\rangle$.
    a. Find $|\vec{v}|$.
    b. Find $2\vec{u} - 5\vec{v}$.
    c. Write $\vec{u}$ in terms of $\vec{\imath}$ and $\vec{\jmath}$.
    d. Find $|-3\vec{u}|$.
    e. Find the direction (in degrees) of $\vec{u}$.
    f. Find $\vec{u} \cdot \vec{v}$.
    g. Find the angle between $\vec{u}$ and $\vec{v}$.

28. Two forces of 19 and 32 newtons act on a point in the plane. If the angle between the forces is $118°$, find the magnitude of the resultant force.

29. Determine whether $\vec{u} = \langle -3,1\rangle$ and $\vec{v} = \langle -2,7\rangle$ are orthogonal.

30. A force of 30.0 lb is required to hold an 80.0-lb pressure washer on an incline. What angle does the incline make with the horizontal?

31. Two forces of 128 lb and 253 lb act at a point. The resultant force is 320 lb. Find the angle between the forces.

32. Solve each quadratic equation and express all nonreal complex solutions in $a + bi$ form.
    a. $2x^2 + 3x = -2$
    b. $x^2 + 2 = 2x$

33. Simplify each power of i.
    a. i^{60}
    b. i^{67}

34. Write $-2 + 2i$ in trigonometric form. Use radians for the angles.

35. Write $-3i$ in trigonometric form. Use radians for the angles.

36. Find the product of $8(\cos 210^\circ + i \sin 210^\circ)$ and $2(\cos 330^\circ + i \sin 330^\circ)$. Write the result in rectangular form.

37. Find the quotient of $8(\cos 210^\circ + i \sin 210^\circ)$ and $2(\cos 330^\circ + i \sin 330^\circ)$. Write the result in rectangular form.

38. Find $\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)^8$ and express the result in rectangular form.

39. Find all cube roots of $2 - 2i\sqrt{3}$. Leave your solutions in trig form with degrees.

40. Find all complex solutions of $x^5 - i = 0$. Leave your solutions in trig form with degrees.

41. For the following polar equation, find an equivalent equation in rectangular coordinates, and graph. Be sure to draw arrows in the direction of increasing θ.
$r = \frac{3}{4\cos\theta - \sin\theta}$

42. For the following polar equation, find an equivalent equation in rectangular coordinates, and graph. Be sure to draw arrows in the direction of increasing θ.
$r = -5\csc\theta$

43. Graph the following polar curves. Be sure to draw arrows in the direction of increasing θ.
    a. $r = 2 - \cos\theta$
    b. $r = 4\sin 2\theta$
    c. $r = 1 - 2\sin\theta$

44. Find a rectangular equation for the following curve, then graph it. Be sure to draw arrows in the direction of increasing t.
$x = 3t + 2,\ y = t - 1,$ for t in $[-2, 1]$

45. Find a rectangular equation for the following curve, then graph it. Be sure to draw arrows in the direction of increasing t.
$x = 2\cos 2t,\ y = 2\sin 2t,$ for t in $[0, 2\pi]$