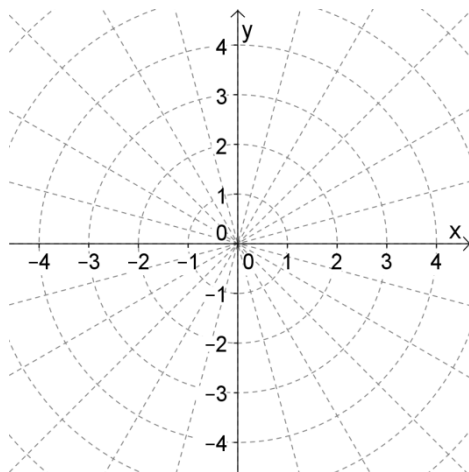


Polar Equations and Graphs

Let's get practice plotting point using polar coordinates (r, θ) :



Notes:

r can be negative (ex: $(1,0) = (-1, \pi)$).

Each (x, y) point has an infinite number of polar coordinate representations (ex: $(1,0) = (1, 2\pi k)$).

Here are the equations to get you from and to polar coordinates:

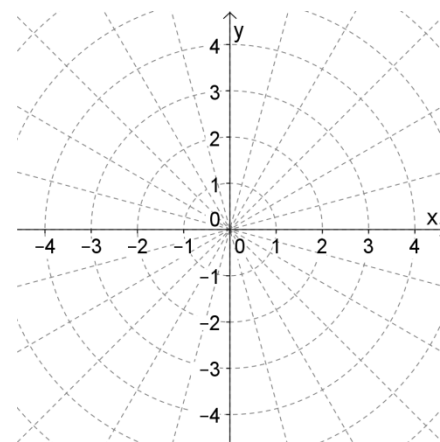
$$x = r \cos \theta, \quad y = r \sin \theta$$

$$r^2 = x^2 + y^2, \quad \tan \theta = \frac{y}{x}$$

Ex 1.

For each of the following pairs of polar coordinates, first plot in the coordinate system to the right, then give two other pairs of polar coordinates, then give the rectangular coordinates.

$(2, 30^\circ)$

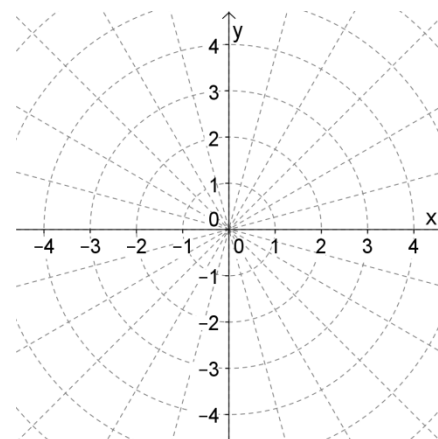


$\left(-4, \frac{2\pi}{3}\right)$

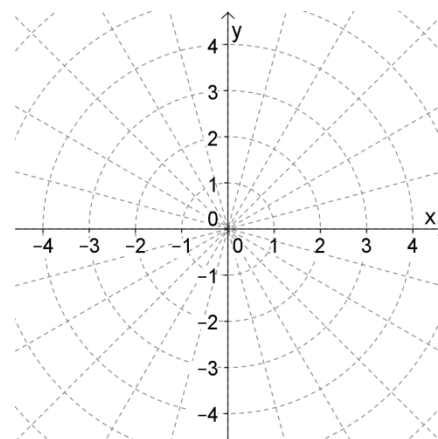
Ex 2.

For the following rectangular equation, give its equivalent polar equation and sketch its graph.

$$y = x - 3$$

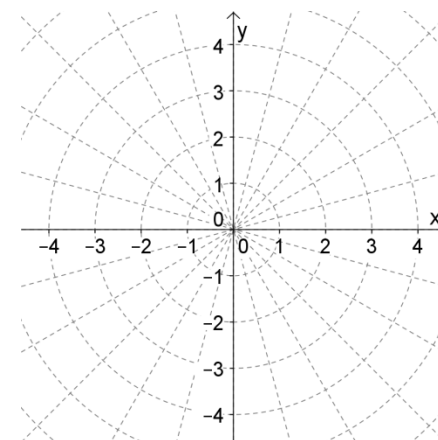


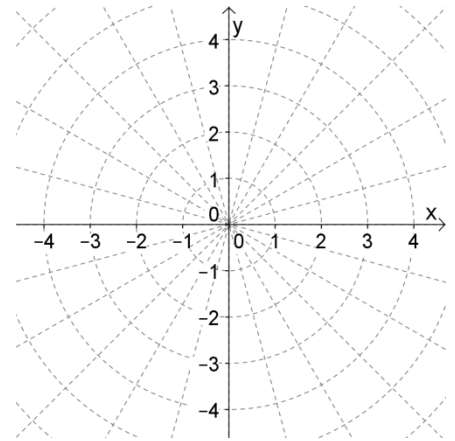
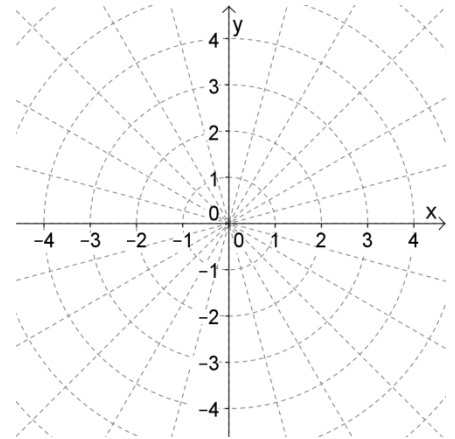
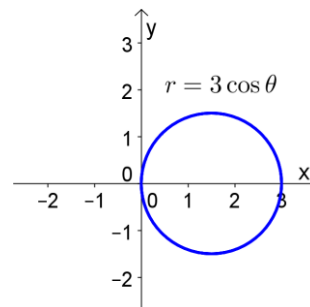
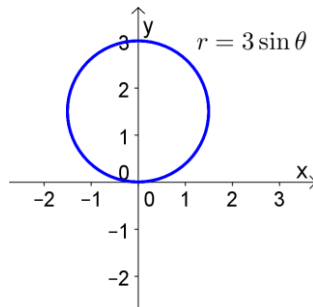
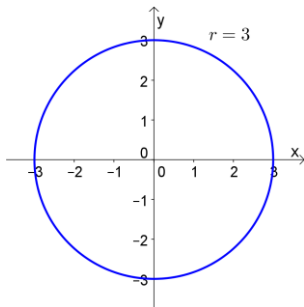
$$x^2 + y^2 = 4$$

**Ex 3.**

For the following polar equation, find an equivalent equation in rectangular coordinates, and graph.

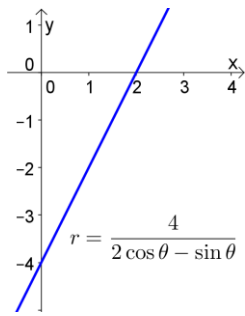
$$r = \frac{4}{2 \cos \theta - \sin \theta}$$



Ex 4.Graph $r = 1 + \cos \theta$.**Ex 5.**Graph $r = 3 \cos 2\theta$.**Common Polar Equations and Graphs****Circle** $r = a$, $r = a \sin \theta$, and $r = a \cos \theta$ 

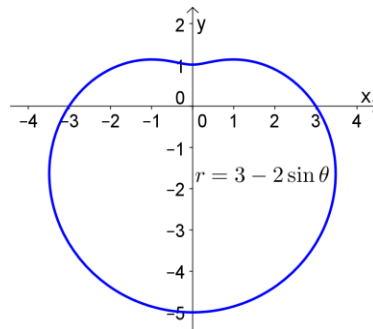
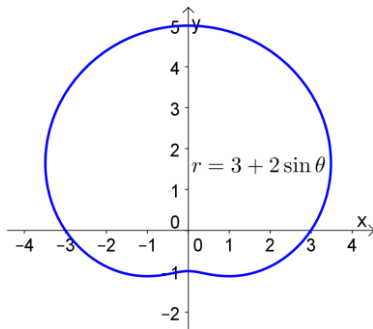
Line

$$r = \frac{c}{a \cos \theta + b \sin \theta}$$

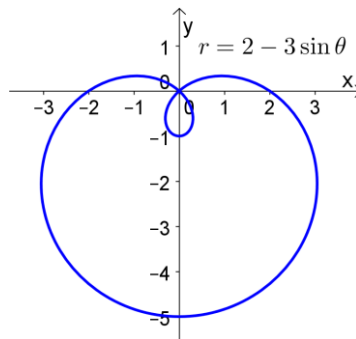
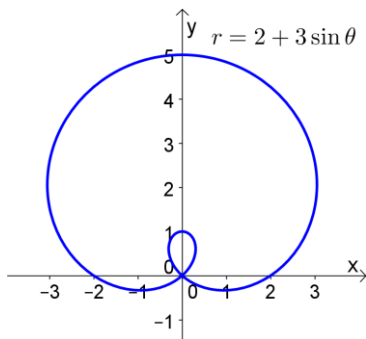
**Limacon** (pronounced “LEE-ma-sahn”)

$$r = a \pm b \sin \theta \text{ and } r = a \pm b \cos \theta$$

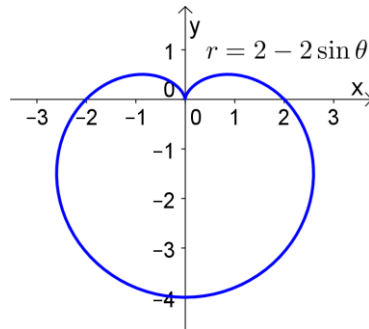
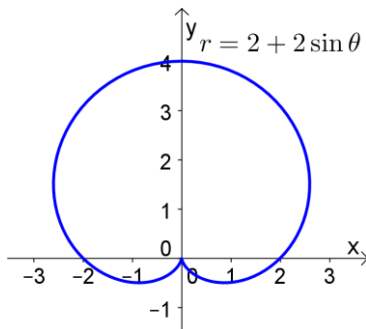
If $a > b$, there is no inner loop.



If $a < b$, there is an inner loop.

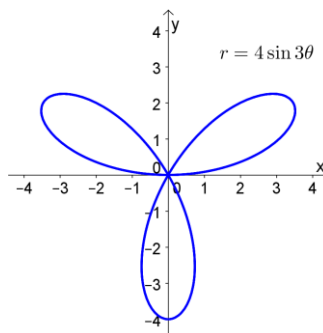
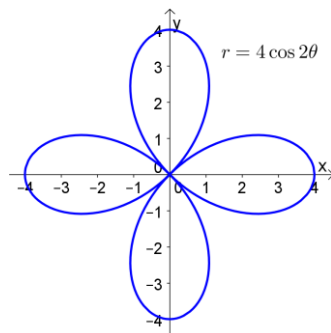
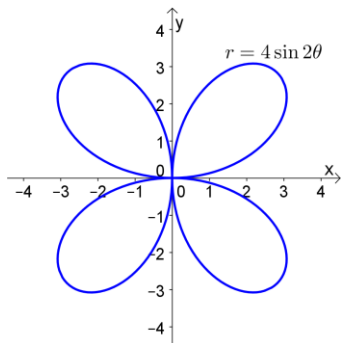


If $a = b$, it's called a cardioid (heart-shaped).

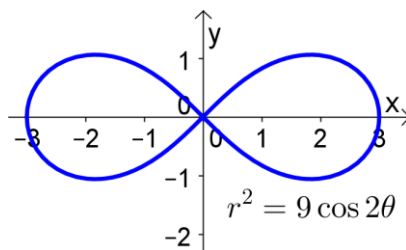
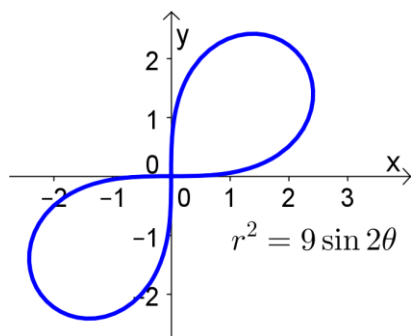


Rose

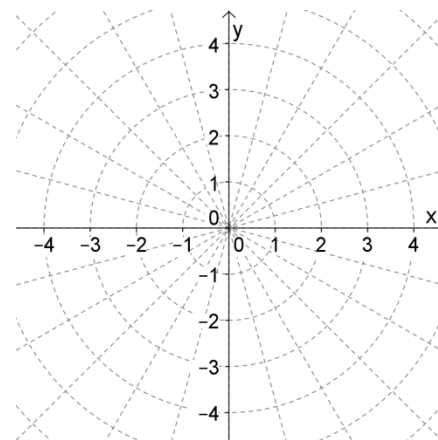
$$r = a \sin n\theta \quad \text{and} \quad r = a \cos n\theta \quad (n \geq 2)$$

**Lemniscate** (rhymes with “biscuit”)

$$r^2 = a^2 \sin 2\theta \quad \text{and} \quad r^2 = a^2 \cos 2\theta$$

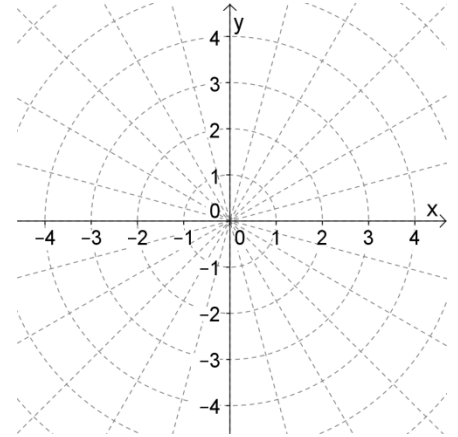
**Practice**

1. For the following pair of polar coordinates, first plot in the coordinate system to the right, then give two other pairs of polar coordinates, then give the rectangular coordinates.
 $(3, 120^\circ)$

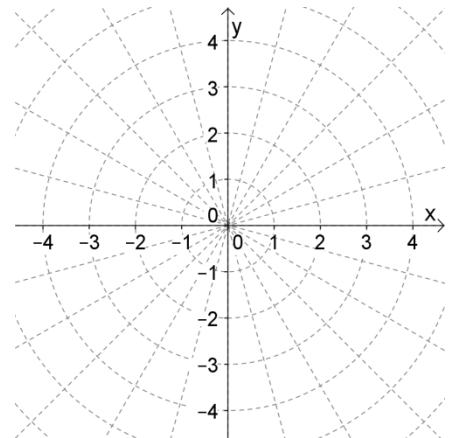


2. For the following polar equation, find an equivalent equation in rectangular coordinates, and graph.

$$r = 2 \cos \theta$$



3. Graph $r = 4 \cos 3\theta$.



Q: What are the next two letters in this sequence: A E F H I K L M ?