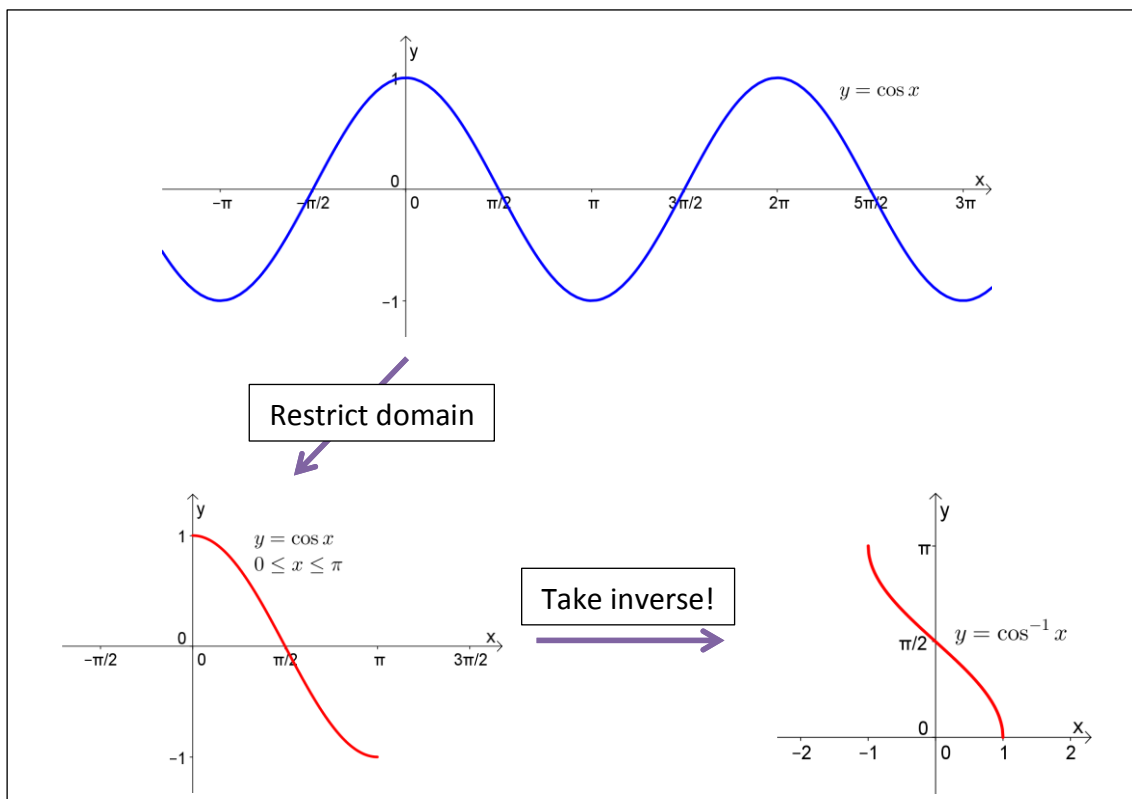
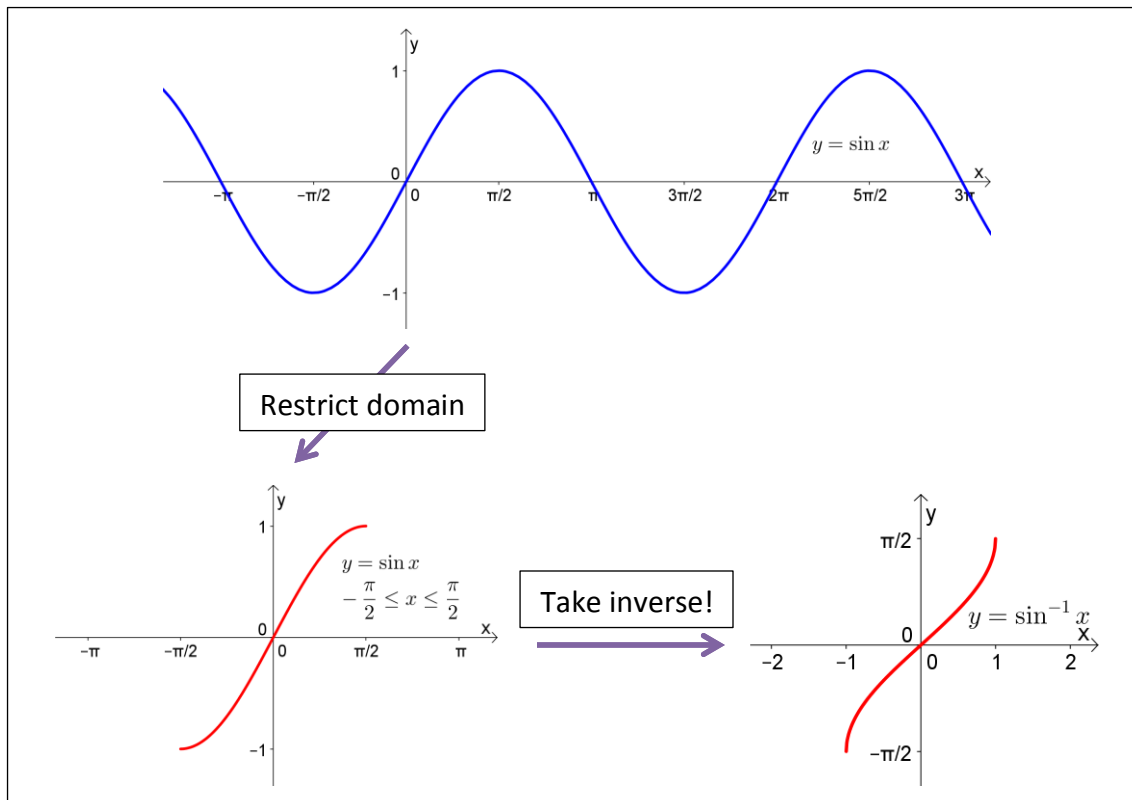
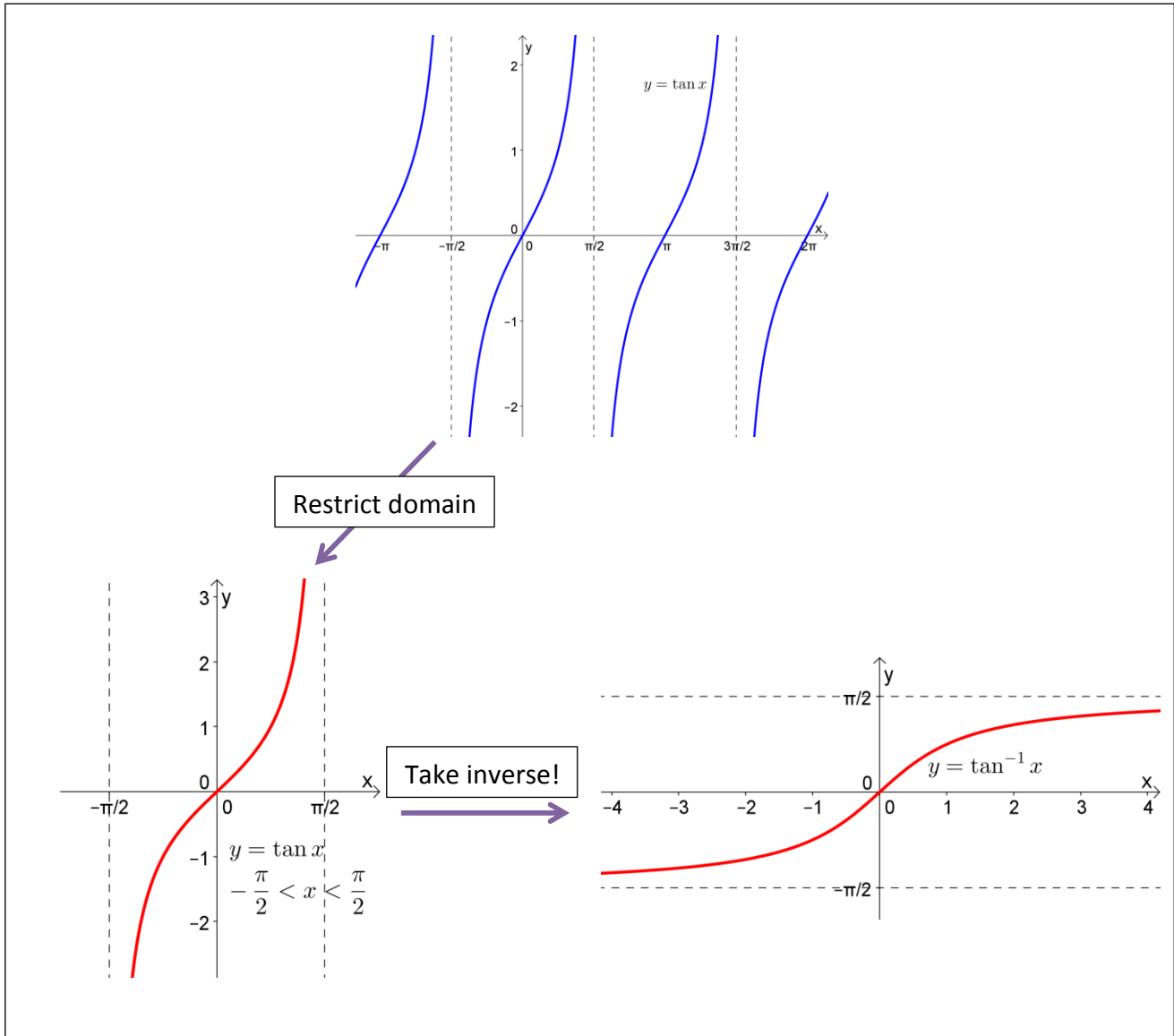


Inverse Circular Functions

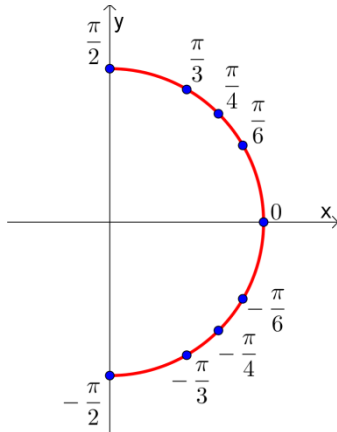
Only one-to-one functions have inverses. So, how can we make inverses of the trig functions? We first need to restrict their domains to make them one-to-one.



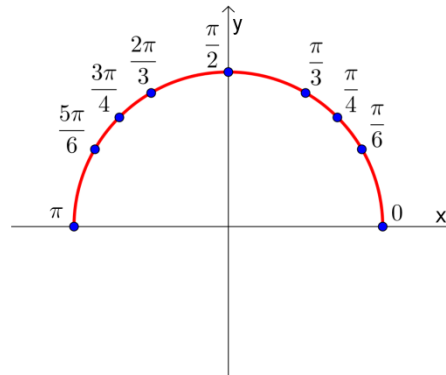


	Domain	Range
$\sin^{-1} x$	$[-1, 1]$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
$\cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1} x$	$(-\infty, \infty)$	$(-\frac{\pi}{2}, \frac{\pi}{2})$

$\sin^{-1} x$ and
 $\tan^{-1} x$ output:



$\cos^{-1} x$ outputs:



Ex 1.

Find the exact value if it exists. Do not use a calculator.

$$\sin^{-1} \frac{1}{2} =$$

$$\sin^{-1} \left(-\frac{1}{2} \right) =$$

$$\sin^{-1} \frac{3}{2} =$$

$$\cos^{-1} \frac{\sqrt{3}}{2} =$$

$$\cos^{-1} 0 =$$

$$\tan^{-1} 1 =$$

$$\tan^{-1} \sqrt{3} =$$

$$\sec^{-1} 2 =$$

$$\cot^{-1} \frac{1}{\sqrt{3}} =$$

Ex 2.

Evaluate the following expression without using a calculator.

$$\tan\left(\cos^{-1}\left(-\frac{5}{13}\right)\right)$$

Ex 3.

Evaluate the following expression without using a calculator.

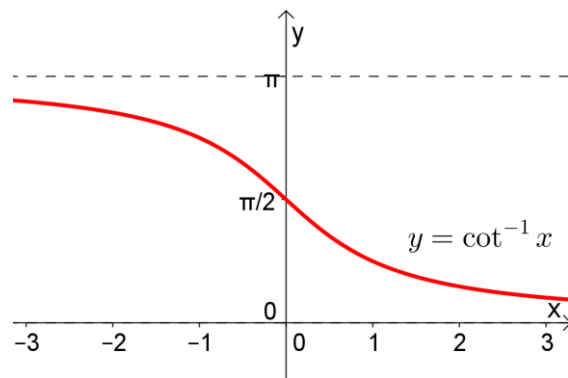
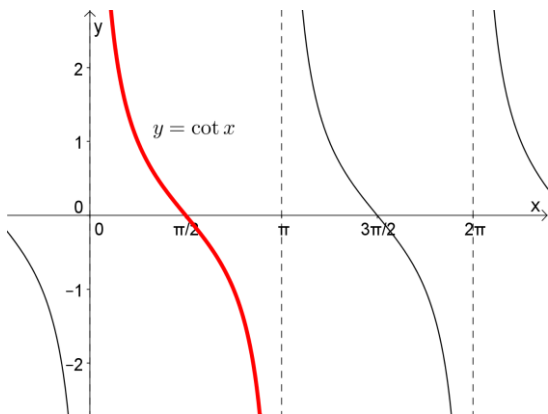
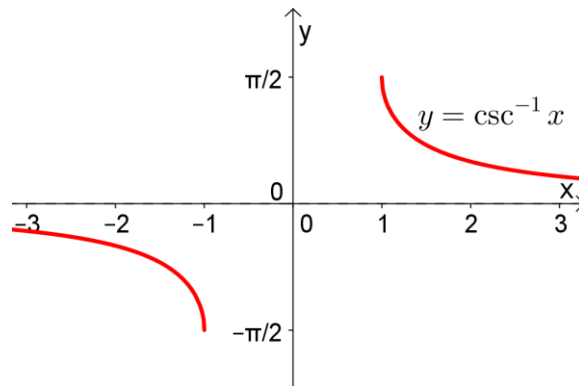
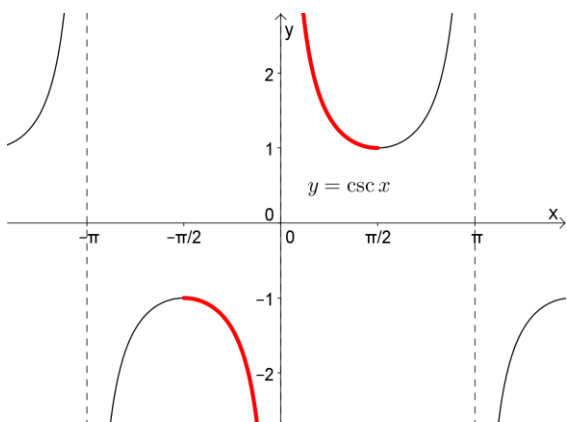
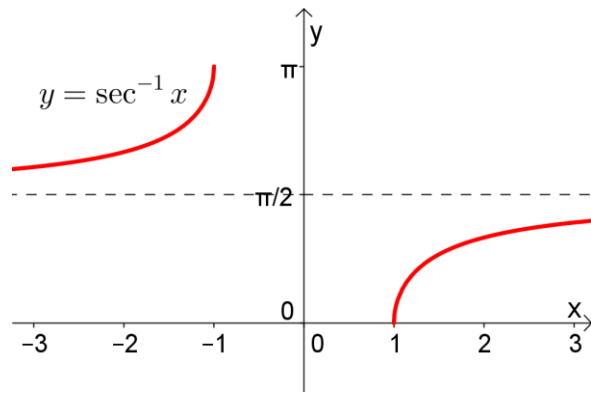
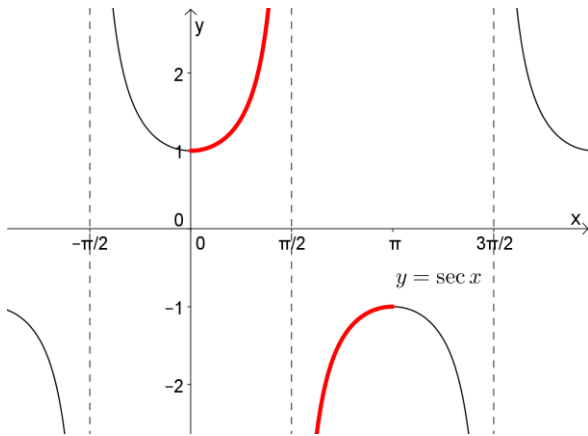
$$\cos\left(\tan^{-1}\sqrt{3} + \sin^{-1}\frac{1}{3}\right)$$

Ex 4.

Rewrite the expression as an algebraic expression in x .

$$\cos(\tan^{-1} x)$$

Here are the other inverse trig functions:



Note:

$$\sec^{-1} x = \cos^{-1} \frac{1}{x}$$

$$\csc^{-1} x = \sin^{-1} \frac{1}{x}$$

$$\cot^{-1} x = \begin{cases} \tan^{-1} \frac{1}{x} & \text{if } x > 0 \\ \pi + \tan^{-1} \frac{1}{x} & \text{if } x < 0 \end{cases}$$

Why is $\sec^{-1} x = \cos^{-1} \frac{1}{x}$?

$\sec^{-1} x = y$
 $\sec y = x$
 $\frac{1}{\cos y} = x$
 $\cos y = \frac{1}{x}$
 $\cos^{-1} \frac{1}{x} = y$
 Thus, $\sec^{-1} x = \cos^{-1} \frac{1}{x}$.

Note about notation:

$$\arcsin x = \sin^{-1} x$$

$$\arccos x = \cos^{-1} x$$

$$\arctan x = \tan^{-1} x$$

$$\operatorname{arccsc} x = \csc^{-1} x$$

$$\operatorname{arcsec} x = \sec^{-1} x$$

$$\operatorname{arccot} x = \cot^{-1} x$$

Practice

1. Find each exact value without using a calculator.

$$\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) =$$

$$\cos^{-1} 1 =$$

$$\tan^{-1}(-1) =$$

$$\tan^{-1}\frac{\sqrt{3}}{3} =$$

$$\csc^{-1}\sqrt{2} =$$

$$\cot^{-1}(-\sqrt{3}) =$$

2. Evaluate the following expression without using a calculator.

$$\sin\left(\tan^{-1}\frac{3}{2}\right)$$

3. Rewrite the expression as an algebraic expression in x .

$$\tan(\cos^{-1} x)$$

4. Write $\cos(2 \sin^{-1} x)$ as an algebraic expression in x .

5. Use a calculator to give $\arcsin 0.81926439$ as a real number value. (Be sure the calculator is in radians mode).

Q: A man leaves home and, after making three left turns, he ends up back at home, and finds two masked men waiting for him. What is happening?