

## Double- and Half-Angle Identities

We can derive formulas for  $\cos 2A$ ,  $\sin 2A$ , and  $\tan 2A$  by using the addition identities:

$$\cos 2A = \cos(A + A) = \cos A \cos A - \sin A \sin A = \cos^2 A - \sin^2 A$$

$$\sin 2A = \sin(A + A) = \sin A \cos A + \cos A \sin A = 2 \sin A \cos A$$

$$\tan 2A = \tan(A + A) = \frac{\tan A + \tan A}{1 - \tan A \tan A} = \frac{2 \tan A}{1 - \tan^2 A}$$

### Double-Angle Identities

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\cos 2A = 1 - 2 \sin^2 A$$

$$\cos 2A = 2 \cos^2 A - 1$$

$$\sin 2A = 2 \sin A \cos A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

### Ex 1.

Given  $\cos \theta = \frac{3}{5}$  and  $\sin \theta < 0$ , find  $\sin 2\theta$ .

### Ex 2.

Find  $\cot \theta$  if  $\cos 2\theta = \frac{4}{5}$  and  $90^\circ < \theta < 180^\circ$ .

**Ex 3.**

Verify that the following equation is an identity.

$$\cot x \sin 2x = 1 + \cos 2x$$

**Ex 4.**

Write  $\cos 3x$  in terms of  $\cos x$ .

We can derive a formula for  $\sin \frac{A}{2}$  by using the double angle identity for  $\cos 2x$ :

$$\cos 2x = 1 - 2 \sin^2 x$$

$$2 \sin^2 x = 1 - \cos 2x$$

$$\sin x = \pm \sqrt{\frac{1 - \cos 2x}{2}}$$

$$\sin \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{2}}$$

### Half-Angle Identities

$$\sin \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$\cos \frac{A}{2} = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$\tan \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}}$$

$$\tan \frac{A}{2} = \frac{\sin A}{1 + \cos A}$$

$$\tan \frac{A}{2} = \frac{1 - \cos A}{\sin A}$$

#### Ex 5.

Find the exact value of  $\tan 22.5^\circ$ .

#### Ex 6.

Given  $\cos x = -\frac{3}{7}$  and  $\pi < x < \frac{3\pi}{2}$ , find  $\sin \frac{x}{2}$ .

**Ex 7.**

Verify that the following equation is an identity.

$$\left(\sin \frac{x}{2} + \cos \frac{x}{2}\right)^2 = 1 + \sin x$$

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**Practice**

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1. Find  $\tan \theta$  if  $\cos 2\theta = \frac{3}{4}$  and  $180^\circ < \theta < 270^\circ$ .

2. Verify that the following equation is an identity.

$$\frac{1 + \cos 2x}{\sin 2x} = \cot x$$

3. Write  $\sin 3x$  in terms of  $\sin x$ .

4. Find the exact value of  $\sin 112.5^\circ$ .

5. Given  $\sin x = -\frac{4}{5}$  and  $\pi < x < \frac{3\pi}{2}$ , find  $\cos \frac{x}{2}$ .

Q: What has many keys but can't open any doors?