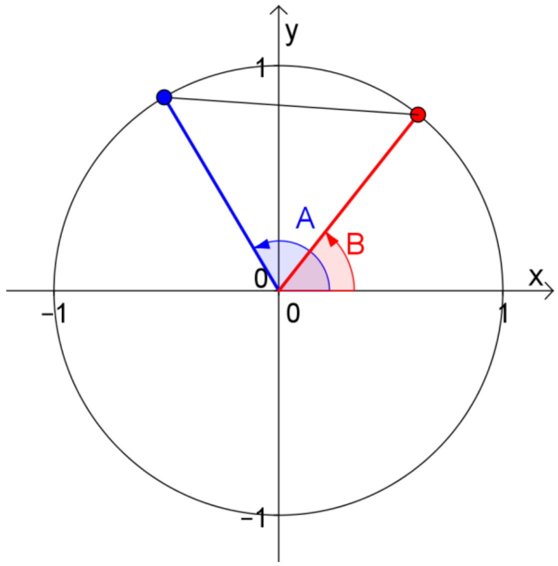
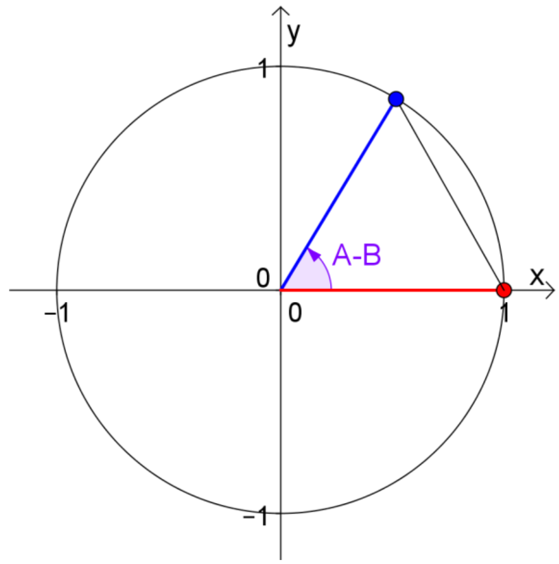
Sum and Difference Identities

Note that $\cos(A-B) \neq \cos A - \cos B$. Take $A = \frac{\pi}{2}$ and $B = 0$ for example.

$\cos(A-B) = \cos\left(\frac{\pi}{2} - 0\right) = \cos\frac{\pi}{2} = \mathbf{0}$ ...but... $\cos A - \cos B = \cos\frac{\pi}{2} - \cos 0 = 0 - 1 = \mathbf{-1}$

So what is $\cos(A-B)$ equal to?





Using the distance formula twice, we get:

$$\sqrt{[\cos(A-B)-1]^2 + [\sin(A-B)-0]^2} = \sqrt{(\cos A - \cos B)^2 + (\sin A - \sin B)^2}$$
$$\cos^2(A-B) - 2\cos(A-B) + 1 + \sin^2(A-B)$$
$$= \cos^2 A - 2\cos A\cos B + \cos^2 B + \sin^2 A - 2\sin A\sin B + \sin^2 B$$
$$2 - 2\cos(A-B) = 2 - 2\cos A\cos B - 2\sin A\sin B$$
$$-2\cos(A-B) = -2\cos A\cos B - 2\sin A\sin B$$
$$\boldsymbol{\cos(A-B) = \cos A\cos B + \sin A\sin B}$$

To find $\cos(A+B)$, we can rewrite it as $\cos(A-(-B))$:

$$\begin{aligned}\cos(A+B) &= \cos\big(A-(-B)\big)\\ &= \cos A\cos(-B) + \sin A\sin(-B)\\ &= \cos A\cos B - \sin A\sin B\end{aligned}$$

To find $\sin(A+B)$, we can make use of $\sin\theta = \cos(90^\circ - \theta)$:

$$\begin{aligned}\sin(A+B) &= \cos\big(90^\circ-(A+B)\big)\\ &= \cos\big((90^\circ-A)-B\big)\\ &= \cos(90^\circ-A)\cos B + \sin(90^\circ-A)\sin B\\ &= \sin A\cos B + \cos A\sin B\end{aligned}$$

To find $\sin(A-B)$, we can rewrite it as $\sin\big(A+(-B)\big)$:

$$\begin{aligned}\sin(A-B) &= \sin\big(A+(-B)\big)\\ &= \sin A\cos(-B) + \cos A\sin(-B)\\ &= \sin A\cos B - \cos A\sin B\end{aligned}$$

Sum/difference identities for tangent can be derived using the sum/difference identities for sine and cosine. So, in ***sum***mary…

Sum/Difference Identities

$\sin(A+B) = \sin A \cos B + \cos A \sin B$
$\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$
$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$
$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

Ex 1.
Use identities to find each exact value.
$\cos 15°$

$\sin \frac{5\pi}{12}$

$\tan \frac{7\pi}{12}$

Ex 2.
Find $\cos(s+t)$ and $\sin(s-t)$ if $\sin s = \frac{3}{5}$ and $\cos t = -\frac{12}{13}$ and both s and t are in QII. Also, which quadrant is $s+t$ in?

Ex 3.
Verify the following identity.
$\frac{\sin(x+y)}{\cos x \cos y} = \tan x + \tan y$

Practice

1. Use identities to find each exact value.

   a) $\sin(-75^\circ)$

   b) $\cos\frac{17\pi}{12}$

2. Write the following as a function of θ and simplify.

   $\sin(180^\circ - \theta)$

3. Find $\sin(s+t)$ if $\sin s = \frac{4}{5}$ and $\cos t = -\frac{5}{13}$ and $\frac{\pi}{2} < s < \pi$ and $\pi < t < \frac{3\pi}{2}$. Also, which quadrant is $s+t$ in?

4. Verify the following identity.
$\sin(x+y) + \sin(x-y) = 2\sin x \cos y$

Q: What has four wheels and flies?