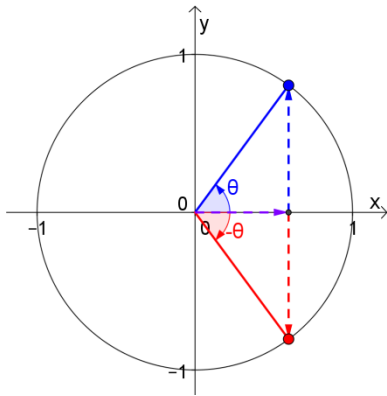


Trigonometric Identities



Even/Odd Identities (also called the Negative-Angle Identities)

$$\begin{aligned} \sin(-\theta) &= -\sin \theta & \cos(-\theta) &= \cos \theta & \tan(-\theta) &= -\tan \theta \\ \csc(-\theta) &= -\csc \theta & \sec(-\theta) &= \sec \theta & \cot(-\theta) &= -\cot \theta \end{aligned}$$

Also recall the following identities:

Reciprocal Identities

$$\sin \theta = \frac{1}{\csc \theta} \quad \csc \theta = \frac{1}{\sin \theta}$$

$$\cos \theta = \frac{1}{\sec \theta} \quad \sec \theta = \frac{1}{\cos \theta}$$

$$\tan \theta = \frac{1}{\cot \theta} \quad \cot \theta = \frac{1}{\tan \theta}$$

Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Quotient Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

Ex 1.

Write $\cos x$ in terms of $\tan x$.

There are two main ways to prove identities:

1. Start from one side (usually the more complicated side), and take steps to get to the other side.

OR

2. Simplify both sides until they look the same.

If you get stuck, convert to sin and cos.

Ex 2.

Verify that the following equation is an identity.

$$1 + \tan \theta = \sec \theta (\sin \theta + \cos \theta)$$

Ex 3.

Verify that the following equation is an identity.

$$\tan^2 x (1 + \cot^2 x) = \frac{1}{1 - \sin^2 x}$$

Ex 4.

Verify that the following equation is an identity.

$$\frac{\tan x - \cot x}{\sin x \cos x} = \sec^2 x - \csc^2 x$$

Ex 5.

Verify that the following equation is an identity.

$$\frac{\cos x}{1 - \sin x} = \frac{1 + \sin x}{\cos x}$$

Ex 6.

Verify that the following equation is an identity.

$$\frac{\sec \alpha + \tan \alpha}{\sec \alpha - \tan \alpha} = \frac{1 + 2 \sin \alpha + \sin^2 \alpha}{\cos^2 \alpha}$$

Practice

1. Write $\cot x$ in terms of $\sin x$.

2. Verify that each equation is an identity.

a) $\cot^2 x (\tan^2 x + 1) = \csc^2 x$

b) $\frac{\tan^2 x}{\sec^2 x} = (1 + \cos x)(1 - \cos x)$

$$c) \frac{\sec x + \tan x}{\sin x} = \frac{\csc x}{\sec x - \tan x}$$

$$d) \frac{1 + \cos \theta}{\cos \theta} = \frac{\tan^2 \theta}{\sec \theta - 1}$$

$$e) \sec x + \tan x = \frac{\cos x}{1 - \sin x}$$

Q: A man rode his horse into town on Tuesday. Two days later he rode home on Tuesday. How is this possible?