Translations of the Graphs of the Sine and Cosine Functions

Here are examples of common transformations on functions (here, the example function is $\sin x$):

$y = 2\sin x$ stretches vertically by factor of 2

$y = \sin 2x$ shrinks horizontally by factor of $\frac{1}{2}$

$y = \sin\left(x - \frac{\pi}{3}\right)$ shifts right by $\frac{\pi}{3}$

$y = 1 + \sin x$ shifts up by 1

Transformations of Functions

For a function $f(x)$, and $c > 0$,

$f(x) + c$ shifts up

$f(x) - c$ shifts down

$cf(x)$ stretches vertically (if $c > 1$), or shrinks vertically (if $0 < c < 1$)

$-f(x)$ reflects about x-axis

$f(x - c)$ shifts right

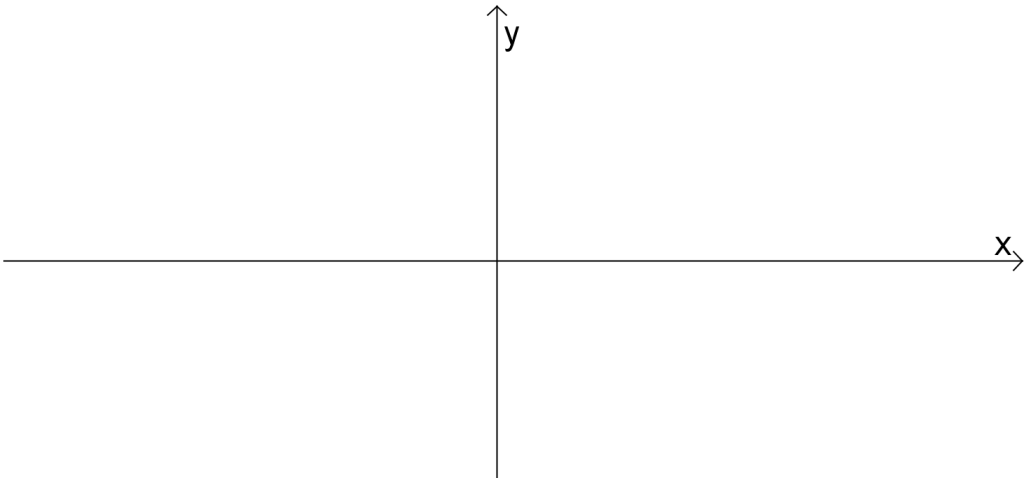
$f(x + c)$ shifts left

$f(cx)$ stretches horizontally (if $0 < c < 1$), or shrink horizontally (if $c > 1$)

$f(-x)$ reflects about y-axis

Ex 1.

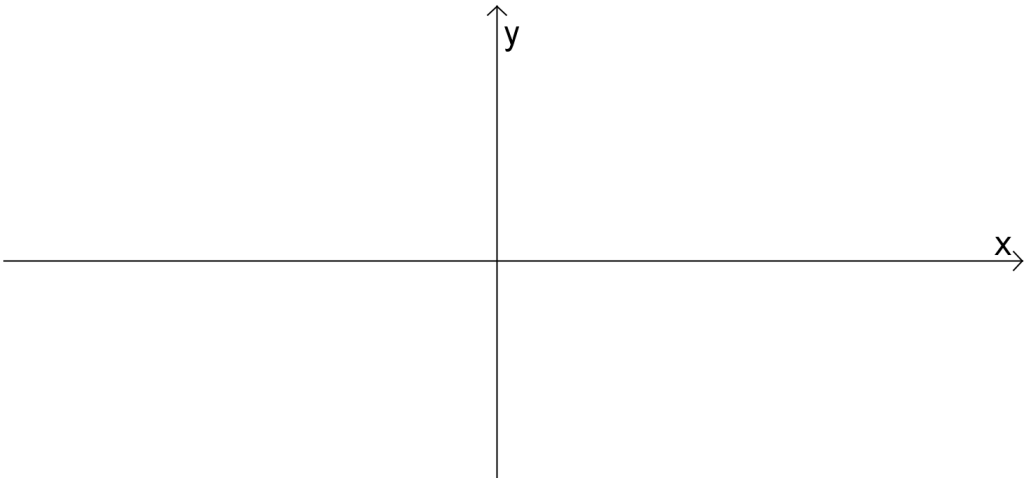
Graph $y = \sin\left(x - \frac{\pi}{3}\right)$.



In the example above, $\frac{\pi}{3}$ is called the _____.

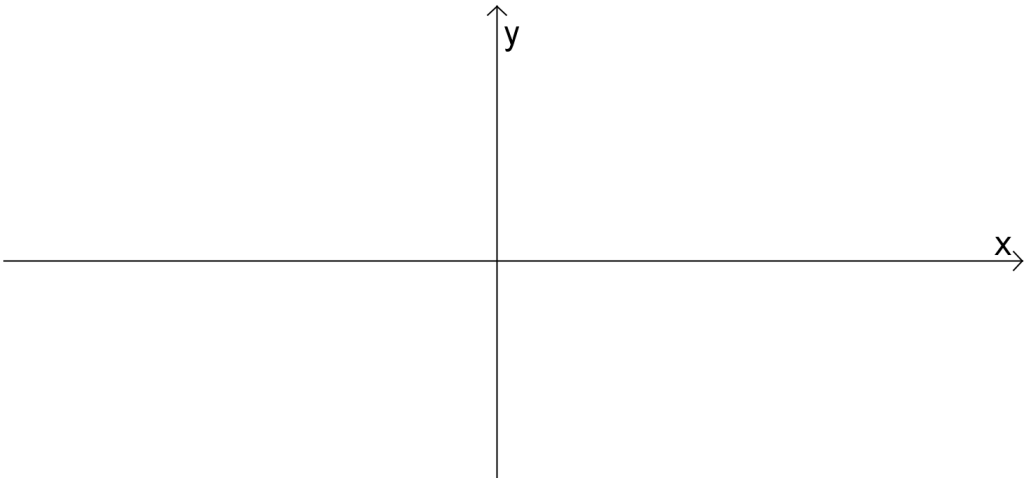
Ex 2.

Graph $y = 3\cos\left(x + \frac{\pi}{4}\right)$ and find the amplitude, period, and phase shift.

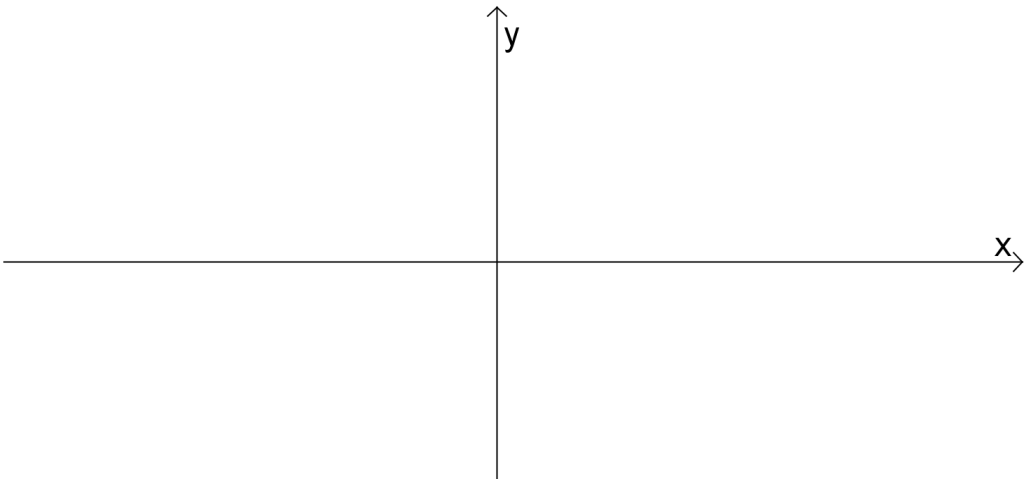


Ex 3.

Graph $y = -2\cos(3x + \pi)$ and find the amplitude, period, and phase shift.

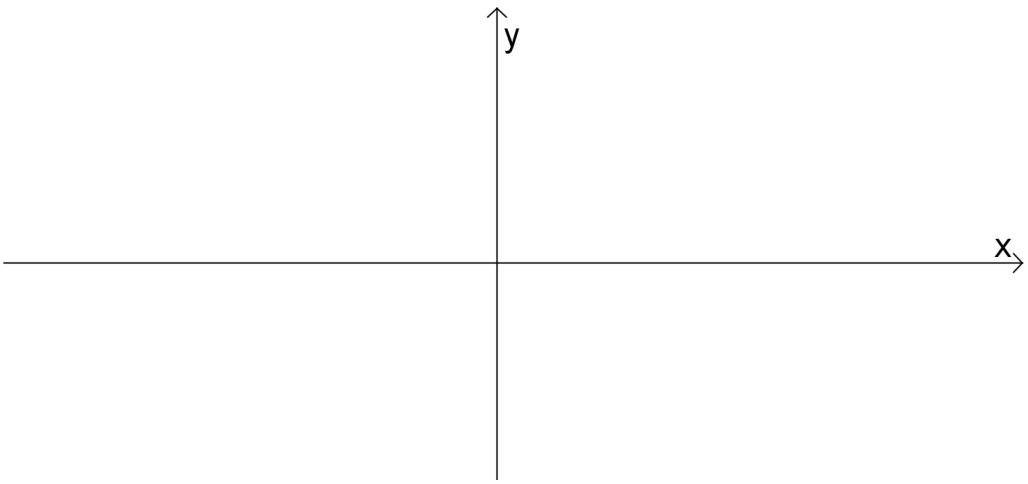


Ex 4.
Graph $y = -1 + 2\sin(4x + \pi)$ and find the amplitude, period, and phase shift.

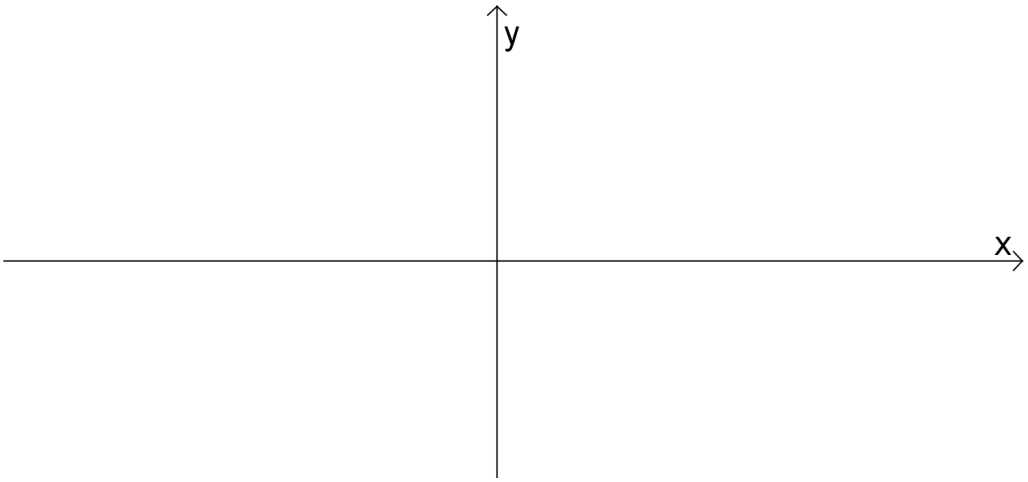


Practice

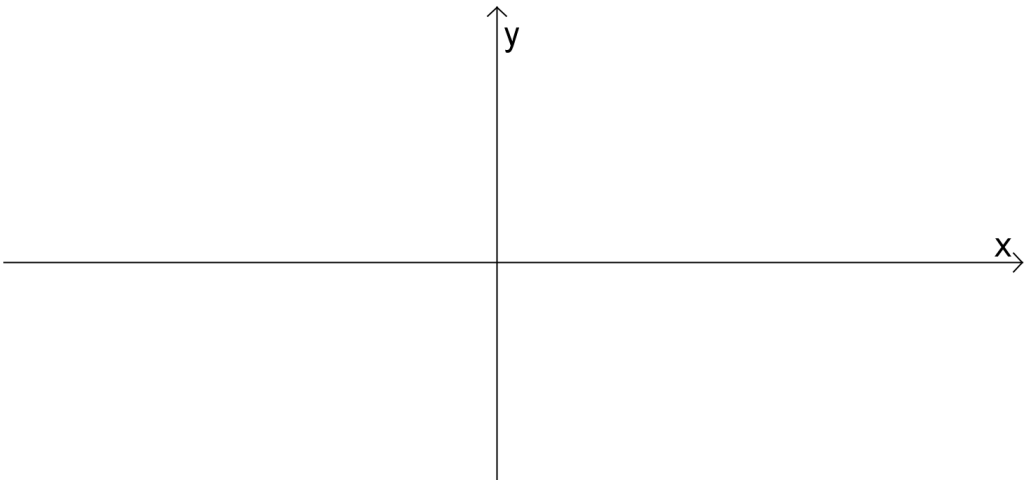
1. Graph $y = 3\sin\left(x - \frac{\pi}{2}\right)$ and find the amplitude, period, and phase shift.



2. Graph $y = -\frac{1}{2}\cos\left(\frac{1}{2}x + \pi\right)$ and find the amplitude, period, and phase shift.



3. Graph $y = 1 - 3\sin\pi\left(x - \frac{1}{3}\right)$ and find the amplitude, period, and phase shift. (Note: if you get this one right, you are a sine-graphing Jedi…)



Q: How can half of 12 be 7?