The Unit Circle and Circular Functions

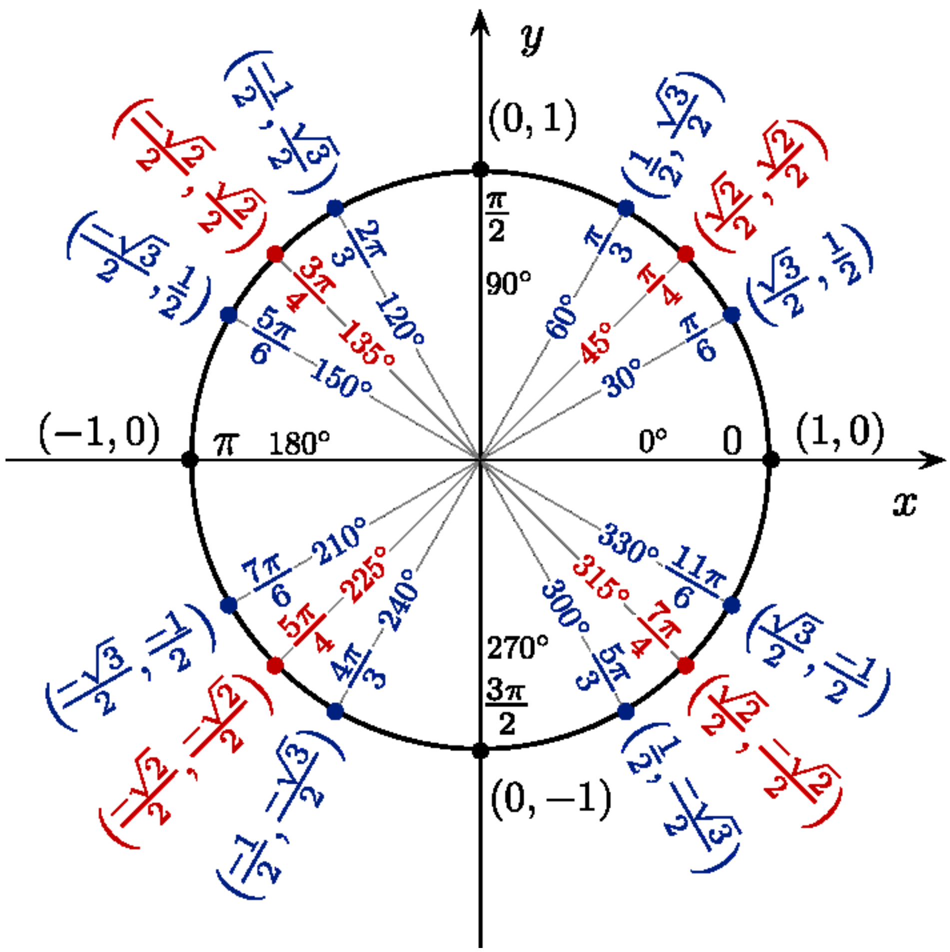
Recall: We've defined the sine function in two ways: $\sin\theta = \frac{y}{r}$ and $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$.

All the trig functions can also be defined in terms of the **unit circle** (circle with radius 1, centered at the origin). Since $s = \theta r$, notice that on the unit circle $s = \theta$. So, angles and arc lengths on the unit circle are the same.

So, the trig functions we learned about in 1.3 become:

$\mathbf{\sin s = y}$	$\mathbf{\csc s = \frac{1}{y}}$
$\mathbf{\cos s = x}$	$\mathbf{\sec s = \frac{1}{x}}$
$\mathbf{\tan s = \frac{y}{x}}$	$\mathbf{\cot s = \frac{x}{y}}$

In particular, note that on the unit circle, x is cosine and y is sine.
Side note: the trig functions are sometimes called the *circular functions*.



Instead of reference angles (from 2.2), we use reference numbers (also called reference arcs).
Reference numbers are the shortest distance along the unit circle to the x-axis.

Ex 1.

Find the exact values of the following.

$\sin\frac{3\pi}{2}$

$\cos\frac{3\pi}{2}$

$\tan\frac{3\pi}{2}$

$\cos\frac{4\pi}{3}$

$\sin\frac{4\pi}{3}$

$\tan\left(-\frac{9\pi}{4}\right)$

Practice

1. Find the exact values of the following.

a) $\cos\frac{5\pi}{4}$

b) $\sin\frac{2\pi}{3}$

c) $\tan\left(-\frac{5\pi}{6}\right)$

d) $\cot\pi$

e) $\csc\frac{13\pi}{3}$

f) $\sec\frac{9\pi}{2}$

Q: What five-letter word becomes shorter when you add two letters to it?