

Trigonometric Functions of Acute Angles

Another way you can define the trig functions is directly from right triangles.

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

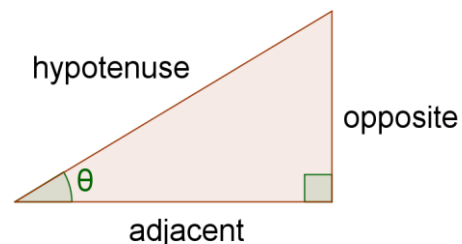
$$\csc \theta = \frac{\text{hyp}}{\text{opp}}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\sec \theta = \frac{\text{hyp}}{\text{adj}}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

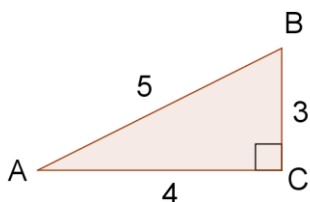
$$\cot \theta = \frac{\text{adj}}{\text{opp}}$$



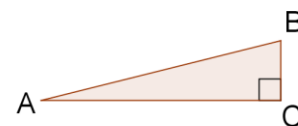
(opp = opposite, adj = adjacent, hyp = hypotenuse)

Ex 1.

Find the sine, cosine, and tangent values for angles A and B in the following right triangle.



Since A and B are complementary angles and $\sin A = \cos B$, sine and cosine are called **cofunctions**. Also, $A + B = 90^\circ$, so $B = 90^\circ - A$, thus $\sin A = \cos(90^\circ - A)$. This is one of the cofunction identities.



Cofunction Identities

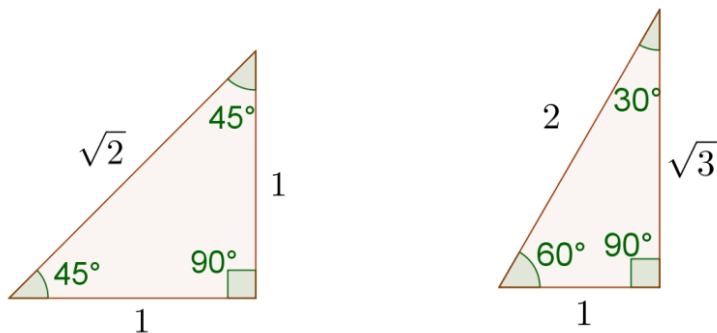
$\sin A = \cos(90^\circ - A)$	$\sec A = \csc(90^\circ - A)$	$\tan A = \cot(90^\circ - A)$
$\cos A = \sin(90^\circ - A)$	$\csc A = \sec(90^\circ - A)$	$\cot A = \tan(90^\circ - A)$

Ex 2.

Find one solution for the following equation. Assume all angles involved are acute angles.

$$\cos(\theta + 4^\circ) = \sin(3\theta + 2^\circ)$$

There are two “special” triangles that can help us evaluate trig functions of 30° , 45° , and 60° angles.

**Ex 3.**

Find the six exact trig function values for a 60° angle.

Note: The word “exact” means not approximated. For example, $\frac{\sqrt{3}}{2}$ is exact, whereas 0.866 (a decimal approximation of $\frac{\sqrt{3}}{2}$) is not exact because it was rounded to the nearest thousandths place. As a general rule, always give exact answers unless otherwise told.

Practice

1. Find one solution to the following equation. Assume all angles involved are acute angles.
 $\cot(\theta - 8^\circ) = \tan(4\theta + 13^\circ)$

2. Suppose ABC is a right triangle with sides of lengths a , b , and c and right angle at C . Find the unknown side length using the Pythagorean Theorem, and then find the values of the six trig function for angle B .
- $a = 3$, $c = 4$

3. Give the exact values of the following expressions.

a) $\sin 45^\circ$

b) $\cot 45^\circ$

c) $\tan 30^\circ$

d) $\csc 60^\circ$

e) $\sec 30^\circ$

f) $\csc 30^\circ$

g) $\cos 30^\circ$