Using the Definitions of the Trigonometric Functions

Recall: An ____________________ is an equation that's always true.

Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta} \quad \csc\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \quad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta} \quad \cot\theta = \frac{1}{\tan\theta}$$

Ex 1.
Find $\tan\theta$ given that $\cot\theta = 4$.

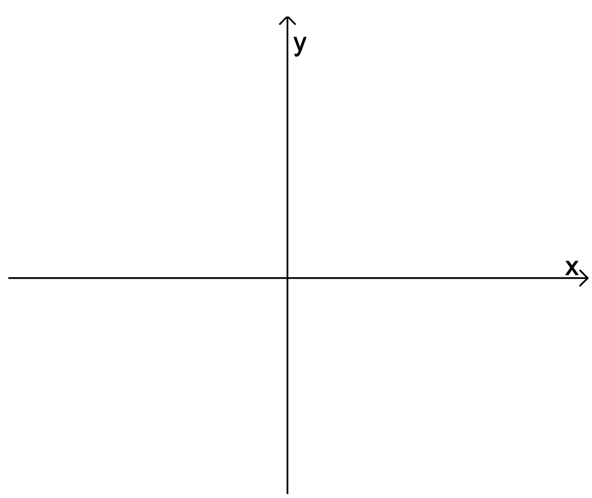
Now, let's continue determining the signs of the trig functions in all 4 quadrants. But first, notice the pairs of trig functions that will always have the same sign.

$$\sin\theta = \frac{y}{r} \quad \csc\theta = \frac{r}{y}$$

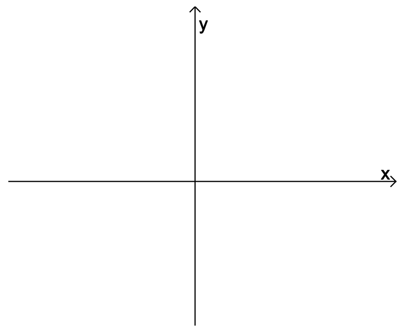
$$\cos\theta = \frac{x}{r} \quad \sec\theta = \frac{r}{x}$$

$$\tan\theta = \frac{y}{x} \quad \cot\theta = \frac{x}{y}$$

So, we can just focus on the signs of $\sin\theta$, $\cos\theta$, and $\tan\theta$.



Here's a quick mnemonic to help remember which trig functions are positive in the quadrants:



Ex 2.
Determine the signs of the trig functions of $260°$.

Ex 3.
Identify the quadrant (or possible quadrants) of an angle θ that satisfies the given conditions.
$\sin\theta < 0,\ \tan\theta > 0$

$\cos\theta > 0,\ \sec\theta > 0$

Trig Function of θ	**Range**
$\sin\theta,\ \cos\theta$	$[-1,1]$
$\tan\theta,\ \cot\theta$	$(-\infty,\infty)$
$\sec\theta,\ \csc\theta$	$(-\infty,-1]\cup[1,\infty)$

Ex 4.
Decide whether each statement is *possible* or *impossible*.
$\cos\theta = -1.7$

$\cot\theta = -1.7$

$\csc\theta = 0$

Ex 5.
Suppose that angle θ is in quadrant II and $\sin\theta = \frac{2}{3}$. Find the values of the other 5 trig functions.

Quotient Identities

$$\boldsymbol{\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}}$$

Proof of $\boldsymbol{\tan\theta = \frac{\sin\theta}{\cos\theta}}$:

$$\frac{\sin\theta}{\cos\theta} = \frac{\left(\frac{y}{r}\right)}{\left(\frac{x}{r}\right)} = \frac{y}{r}\cdot\frac{r}{x} = \frac{y}{x} = \tan\theta$$

Pythagorean Identities

$$\boldsymbol{\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta}$$

Proof of $\boldsymbol{\sin^2\theta + \cos^2\theta = 1}$:
From the Pythagorean Theorem, $x^2 + y^2 = r^2$. Divide both sides by r^2 to get:

$$\frac{x^2}{r^2} + \frac{y^2}{r^2} = \frac{r^2}{r^2}$$

This is the same as:

$$\left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 = 1$$

Since $\cos\theta = \frac{x}{r}$ and $\sin\theta = \frac{y}{r}$, we get:

$$(\cos\theta)^2 + (\sin\theta)^2 = 1$$

Using shorthand, we can write:

$$\boldsymbol{\sin^2\theta + \cos^2\theta = 1}$$

Ex 6.

Find $\sin\theta$ and $\tan\theta$, given that $\cos\theta = -\frac{\sqrt{3}}{4}$ and $\sin\theta > 0$.

Practice

1. Find $\sec\theta$ given that $\cos\theta = -\frac{2}{\sqrt{20}}$.

2. Determine the signs of the trig functions of $-200°$. (Hint: first get a nice coterminal angle!) ☺

3. Decide whether each statement is *possible* or *impossible*.

   a) $\sin\theta = 3$

   b) $\sec\theta = 100$

4. Suppose that angle θ is in quadrant III and $\tan\theta = \frac{8}{5}$. Find the values of the other 5 trig functions.

5. Find $\sin\theta$ and $\cos\theta$, given that $\tan\theta = \frac{4}{3}$ and θ is in quadrant III.