

Quiz #2

Name: _____

Math 140, Prof. Beydler

Wednesday, October 12, 2016

Directions: Show all work. No books or notes. A **scientific calculator** is allowed. Your desk and lap must be clear (no phones, notebooks, etc.). Write your answers in the indicated places, or box your answers. Good luck!

1. (3 points) Find the intervals of increase and decrease for $f(x) = \frac{1}{4-x^2}$

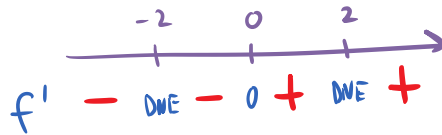
Increasing: $0 < x < 2$ and $x > 2$

Decreasing: $x < -2$ and $-2 < x < 0$

$$f'(x) = \frac{(4-x^2) \cdot 0 - 1 \cdot (-2x)}{(4-x^2)^2} = \frac{2x}{(4-x^2)^2}$$

$$\begin{aligned} \underline{f'(x)=0}: \\ 2x &= 0 \\ x &= 0 \end{aligned}$$

$$\begin{aligned} \underline{f'(x) \text{ DNE}}: \\ (4-x^2)^2 &= 0 \\ 4-x^2 &= 0 \\ 4 &= x^2 \\ x &= \pm 2 \end{aligned}$$



2. (3 points) Find the critical points of $f(x) = 2x + 1 + \frac{2}{x}$ and use the 2nd Derivative Test to classify each critical point as a relative maximum or relative minimum.

Critical points (label each as max or min): Max (-1, -3), Min (1, 5)

$$f'(x) = 2 - \frac{2}{x^2}$$

$$f''(x) = \frac{4}{x^3}$$

$$f''(-1) = \frac{4}{(-1)^3} = -4 < 0 \quad \text{max}$$

$$f(-1) = 2(-1) + 1 + \frac{2}{(-1)} = -3$$

$$\underline{f'(x)=0}:$$

$$2 - \frac{2}{x^2} = 0$$

$$2 = \frac{2}{x^2}$$

$$2x^2 = 2$$

$$x^2 = 1$$

$$x = 1, x = -1$$

$$f''(1) = \frac{4}{1^3} = 4 > 0 \quad \text{min}$$

$$f(1) = 2(1) + 1 + \frac{2}{1} = 5$$

3. (0.5 points) Find the vertical asymptote(s) of $f(x) = \frac{2x^2}{x^2-1}$

$$x^2 - 1 = 0$$

Vertical asymptote(s): $x=1, x=-1$

4. (0.5 points) Find the horizontal asymptote of $f(x) = \frac{2x^2}{x^2-1}$

$$\lim_{x \rightarrow \pm\infty} \frac{2x^2}{x^2-1} = \lim_{x \rightarrow \pm\infty} \frac{2}{1-\frac{1}{x^2}} = \frac{2}{1-0} = 2$$

Horizontal asymptote: $y=2$

5. (2 points) Determine the intervals of concavity for $f(x) = \frac{1}{3}x^3 - 9x + 2$.

$$f'(x) = x^2 - 9$$

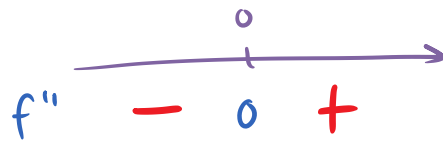
Concave up: $x > 0$

$$f''(x) = 2x$$

Concave down: $x < 0$

$$\frac{f''(x) = 0:}{2x = 0} \quad \frac{f''(x) \text{ DNE:}}{\text{Nowhere}}$$

$$x = 0$$



6. For the function $f(x) = 3x^4 + 4x^3$, do the following:

- (0.5 points) Find the x -intercepts of f .
- (0.5 points) Find the y -intercept of f .
- (2 points) Find $f'(x)$ and $f''(x)$, and determine where each are 0 or do not exist (DNE).
- (1 point) Do a sign analysis on f' and f'' .
- (1 point) Find any maxima and minima, as well as any inflection points.
- (1 point) Sketch the graph of f .

a. $3x^4 + 4x^3 = 0 \rightarrow x^3(3x+4) = 0 \rightarrow x=0, -\frac{4}{3}$

b. $f(0) = 0$

c. $f'(x) = 12x^3 + 12x^2 = 12x^2(x+1)$

$f'(x) = 0: 12x^2(x+1) = 0 \rightarrow x=0, -1$

$f'(x)$ DNE: Nowhere

$f''(x) = 36x^2 + 24x = 12x(3x+2)$

$f''(x) = 0: 12x(3x+2) = 0 \rightarrow x=0, -\frac{2}{3}$

$f''(x)$ DNE: Nowhere

