Math 140 – Final Exam Review Exercises Answer Key

Note: Where possible, I wrote brief descriptions of reasoning—hopefully this provides a little more than just an answer. Please let me know if you find any mistakes or have any questions! - Prof. Beydler

1.

$\lim_{x\to 4^-} f(x) = 2$

$\lim_{x\to 4^+} f(x) = 3$

$\lim_{x\to 4} f(x)$ DNE

2.

$\lim_{x\to 1^-} f(x) = 2$	$\lim_{x\to 1^+} f(x) = 2$	$\lim_{x\to 1} f(x) = 2$
$\lim_{x\to 2^-} f(x) = 2$	$\lim_{x\to 2^+} f(x) = 2$	$\lim_{x\to 2} f(x) = 2$
$\lim_{x\to 3^-} f(x) = 2$	$\lim_{x\to 3^+} f(x) = 1$	$\lim_{x\to 3} f(x)$ DNE
$\lim_{x\to 4^-} f(x) = 3$	$\lim_{x\to 4^+} f(x) = 3$	$\lim_{x\to 4} f(x) = 3$
$\lim_{x\to 6^-} f(x) = 1$	$\lim_{x\to 6^+} f(x) = 0$	$\lim_{x\to 6} f(x)$ DNE
$\lim_{x\to +\infty} f(x) = +\infty$	$\lim_{x\to -\infty} f(x) = 1$	

3.

a. 2 (first, factor top/bottom)

b. $\frac{1}{4}$ (first, divide top/bottom by x^3)

c. $\frac{1}{2}$ (first step is multiply top/bottom by $x + \sqrt{x}$)

d. DNE

4. $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

5. $\frac{1}{2\sqrt{x}}$

6. $-\frac{1}{x^2}$

7. $x = 0, -\frac{1}{2}$

8. $x = -1, 0, 4$

9.

a. $-\frac{3}{(x^2-3)^{3/2}}$

b. $\frac{2(x^2+1)}{2x-3} + 2x\ln(2x-3)$

c. $\frac{x-e^{-2x}}{\sqrt{e^{-2x}+x^2}}$

d. $2(2x+1)^4(x-2)^5(11x-7)$

10. $y = -\frac{4}{9}x - \frac{4}{3}$

11. $y - e = \frac{e}{2}(x-1)$ (or $y = \frac{e}{2}x + \frac{e}{2}$)

12.

   a. $C'(x) = \frac{1}{2}x + 3$ and $R'(x) = 9 - \frac{2}{5}x$
   b. Estimate: $\$11$. Actual: $\$11.25$.
   c. Estimate: $\$2.60$. Actual: $\$2.40$. (Wow, this product is really not good for business!)

13.

   a. $2,200 per year
   b. 4.01% per year

14. $\frac{1-2y}{2x+2y-1}$

15. -4 (Note: $\frac{dy}{dx} = \frac{xy^2}{1-x^2y}$)

16. 15.42 units per month. (Notes: $\frac{dx}{dt} = \frac{-17p\frac{dp}{dt}}{75x}$ and $x \approx 7.72$)

17. 50 bottles per month. (Notes: $\frac{dx}{dt} = \frac{2p\frac{dp}{dt}}{5x}$ and $x = 2$)

18. Increasing: $x < \frac{1}{2}$ Decreasing: $x > \frac{1}{2}$ (Note: $f'(x) = e^{-2x}(1-2x)$)

19. Increasing: $x < 0$ and $x > 6$ Decreasing: $0 < x < 3$ and $3 < x < 6$ (Note: $f'(x) = \frac{x(x-6)}{(x-3)^2}$)

20. Concave up: $x > 1$ Concave down: $x < 1$ Inflection point: $\left(1, \frac{1}{e^2}\right)$ (Note: $f''(x) = 4e^{-2x}(x-1)$)

21. Concave up: $x > 16$ Concave down: $x < -2$ and $-2 < x < 16$ Inflection point: $\left(16, \frac{1}{27}\right)$ (Note: $f'(x) = \frac{10-x}{(x+2)^3}$ and $f''(x) = \frac{2(x-16)}{(x+2)^4}$)

22.

a. $f(0) = \boxed{-2}$

b. $f'(x) = 3x^2 + 6x = 3x(x+2)$

$f''(x) = 6x + 6 = 6(x+1)$

$f' = 0$: $3x(x+2) = 0$ → $x = 0$, $x = -2$

f' DNE: Nowhere

$f'' = 0$: $6(x+1) = 0$ → $x = -1$

f'' DNE: Nowhere

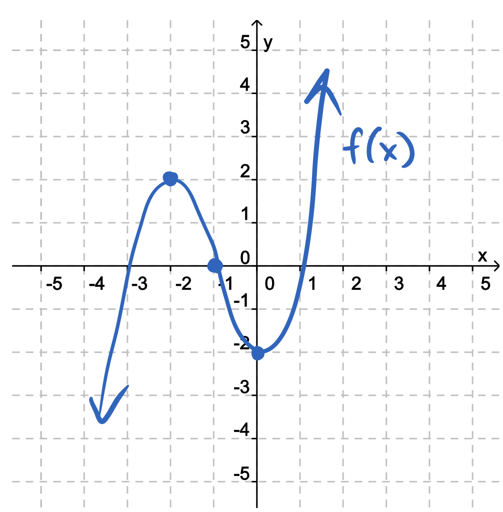
c.

		-2		-1		0	
f'	+	0	−	−	−	0	+
f''	−	−	−	0	+	+	+

d.

Max	I.P.	Min.
$(-2, 2)$	$(-1, 0)$	$(0, -2)$

e.



23.

a. $x^2-4=0 \rightarrow x=\pm 2$

Domain: All real #'s except 2 and −2.

b. $f(0)=\dfrac{0^2+2}{0^2-4}=\boxed{-\dfrac{1}{2}}$

c. H.A:

$$\lim_{x\to\pm\infty}\frac{x^2+2}{x^2-4}=\lim_{x\to\pm\infty}\frac{1+2/x^2}{1-4/x^2}=1 \qquad \boxed{y=1}$$

d. $$f'(x)=\frac{(x^2-4)(2x)-(x^2+2)(2x)}{(x^2-4)^2}=\frac{2x\left((x^2-4)-(x^2+2)\right)}{(x^2-4)^2}=\frac{-12x}{(x^2-4)^2}$$

$$f''(x)=\frac{(x^2-4)^2(-12)-(-12x)\cdot 2(x^2-4)\cdot 2x}{\left[(x^2-4)^2\right]^2}$$

$$=\frac{-12(x^2-4)\left[(x^2-4)-4x^2\right]}{(x^2-4)^{4\,3}}$$

$$=\frac{-12(-3x^2-4)}{(x^2-4)^3}$$

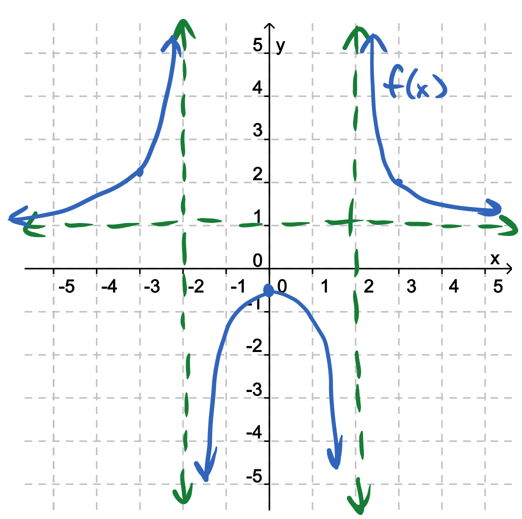
$$=\frac{12(3x^2+4)}{(x^2-4)^3}$$

$f'(x)=0$:	$f'(x)$ DNE:	$f''(x)=0$:	$f''(x)$ DNE:
$-12x=0$	$x^2-4=0$	$12(3x^2+4)=0$	$x^2-4=0$
$x=0$	$x=\pm 2$	No sol.	$x=\pm 2$

e.

x		−2		0		2	
f'	+	DNE	+	0	−	DNE	−
f''	+	DNE	−	−	−	DNE	+

f. Max $(0, -\frac{1}{2})$

g. 

24.

a. Absolute maximum: 2 Absolute minimum: -56 (Note: $f'(x) = 15x^2(x-1)(x+1)$)

b. Absolute maximum: $\frac{81}{10}$ Absolute minimum: 0 (Note: $f'(x) = x^3(x-2)^2$)

c. Absolute minimum: 1 (Notes: $f'(x) = \frac{x-1}{x}$ and use Second Derivative Test)

25.

a. $f'(x) = (x+1)^3(6-x)^2\sqrt[3]{2x+1}\left[\frac{3}{x+1} - \frac{2}{6-x} + \frac{2}{3(2x+1)}\right]$

b. $f'(x) = \frac{\sqrt{2x+3}}{e^{4x}(x-2)^5}\left(\frac{1}{2x+3} - 4 - \frac{5}{x-2}\right)$

26.

a. $-e^{-x} + \frac{2}{5}x^{\frac{5}{2}} - x + \frac{x^{100}}{100} + 5\ln|x| + C$

b. $\frac{1}{2}\ln|x^2 + 4x + 2| + C$ (Note: substitution with $u = x^2 + 4x + 2$)

c. $\frac{1}{2}e^{x^2} + C$ (Note: substitution with $u = x^2$)

d. $\frac{(\ln 2)^3}{3}$ (Note: substitution with $u = \ln x$)

e. $\frac{2}{3}xe^{3x} - \frac{2}{9}e^{3x} + C$ (Note: integration by parts with $u = 2x$ and $dv = e^{3x}dx$)

f. $\frac{x^2}{2}\ln(2x) - \frac{x^2}{4} + C$ (Note: integration by parts with $u = \ln(2x)$ and $dv = xdx$)

27.

a. $f(x) = x^2 + x + 1$

b. $f(x) = \frac{1}{3}\ln|1 + 3x^2| + 5$ (Note: integrate $\frac{2x}{1+3x^2}$ with substitution $u = 1 + 3x^2$)

28.

a. $\sqrt{y^2+1} = \ln|x| + \sqrt{5}$ (or $y = \pm\sqrt{\left(\ln|x| + \sqrt{5}\right)^2 - 1}$)

b. $\frac{y^3}{3} = 2x + \frac{2}{3}x^{3/2} + 9$ (or $y = \sqrt[3]{6x + 2x^{3/2} + 27}$)

29.

a. $\frac{101}{60}$

b. $\frac{73}{45}$

30.

a. Diverges (first step looks like $\lim_{N\to+\infty}\int_0^N (1 + e^{-2x})\,dx$)

b. $\frac{1}{6}$ (first step looks like $\lim_{N\to+\infty}\int_1^N \frac{x^2}{(x^3+1)^2}\,dx$, then do substitution with $u = x^3 + 1$)

31.

a. 1 (integral looks like $\int_0^1 (x - (-x))\,dx$)

b. 9 (integral looks like $\int_{-2}^1 [(-x^2 + 4) - (x^2 + 2x)]\,dx$)

32. $D(4) = \$66.52$, Consumer surplus is \$16.62. (integral for C.S. looks like $\int_0^4 (75e^{-0.03q} - 66.52)\,dq$)

33. $S(4) = \$34.80$, Producer surplus is \$12.80. (integral for P.S. looks like $\int_0^4 (34.8 - (0.3q^2 + 30))\,dq$)

34.

a. Domain: All (x, y) such that $y \le x^2$. $f(3,5) = 2$.

b. Domain: All (x, y) such that $x \neq \pm 1$. $f(0,1) = -1$.

35. $f_x = xye^{xy} + e^{xy} + 5 \quad f_y = x^2 e^{xy}$

36.

a. $f_x = 3e^{3x} + 3y^2 \quad f_y = 6xy + \frac{1}{y} \quad f_{xx} = 9e^{3x} \quad f_{xy} = 6y \quad f_{yx} = 6y \quad f_{yy} = 6x - \frac{1}{y^2}$

b. $f_x = 8(2x - 3y)^3 \quad f_y = -12(2x - 3y)^3 \quad f_{xx} = 48(2x - 3y)^2 \quad f_{xy} = -72(2x - 3y)^2 \quad f_{yx} = -72(2x - 3y)^2 \quad f_{yy} = 108(2x - 3y)^2$

37.

a. $\frac{\partial D_1}{\partial p_2} = 4 > 0$ and $\frac{\partial D_2}{\partial p_1} = 2 > 0$ so commodities are substitutes.

b. $\frac{\partial D_1}{\partial p_2} = -20 < 0$ and $\frac{\partial D_2}{\partial p_1} = -100 < 0$ so commodities are complementary.

38.

a. Max: $(2, 0)$ Saddle point: $(-2, 0)$ (Note: $D(x, y) = 48x$)

b. Min: $(0, 0)$ Saddle points: $(\pm\sqrt{2}, -1)$ (Note: $D(x, y) = 4 + 4y - 4x^2$)

c. Max: $(-2, 0)$ Min: $(0, 2)$ Saddle points: $(0, 0)$ and $(-2, 2)$ (Note: $D(x, y) = 36(x + 1)(y - 1)$)

39.

a. Max value: $\frac{3}{2}$ Min value: -3 (Note: system of equations looks like $\begin{cases} 2x = 2x\lambda \\ -2y - 2 = 2y\lambda \\ x^2 + y^2 = 1 \end{cases}$ and points are $(0, \pm 1)$ and $\left(\pm\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$)

b. Max value: 12 Min value: 3 (Note: system of equations looks like $\begin{cases} 2x + 2 = 2x\lambda \\ 4y = 2y\lambda \\ x^2 + y^2 = 4 \end{cases}$ and points are $(\pm 2, 0)$ and $\left(1, \pm\sqrt{3}\right)$)

40.

a. 32

b. $\frac{e}{2} - 1$