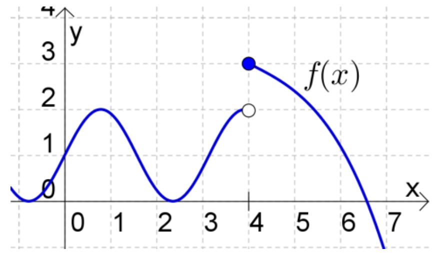
Math 140 – Final Exam Review Exercises

Note: Two or three of these problems will be on the final exam. It is important to understand that these review exercises are <u>not guaranteed to cover all</u> of the potential problems on the final exam. Please review the notes, practice problems, previous quizzes/tests, and homework problems to fully prepare for the final exam. Now, take a deep breath, and get to it! ☺

1. Based on the graph of $f(x)$ shown to the right, find each of the following:

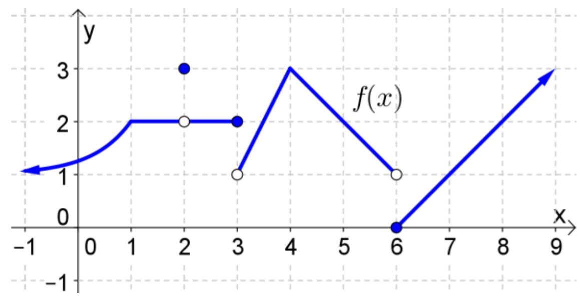


$\lim_{x\to 4^-} f(x)$

$\lim_{x\to 4^+} f(x)$

$\lim_{x\to 4} f(x)$

2. Based on the graph of $f(x)$ shown to the right, find each of the following:



$\lim_{x\to 1^-} f(x)$	$\lim_{x\to 1^+} f(x)$	$\lim_{x\to 1} f(x)$
$\lim_{x\to 2^-} f(x)$	$\lim_{x\to 2^+} f(x)$	$\lim_{x\to 2} f(x)$
$\lim_{x\to 3^-} f(x)$	$\lim_{x\to 3^+} f(x)$	$\lim_{x\to 3} f(x)$
$\lim_{x\to 4^-} f(x)$	$\lim_{x\to 4^+} f(x)$	$\lim_{x\to 4} f(x)$
$\lim_{x\to 6^-} f(x)$	$\lim_{x\to 6^+} f(x)$	$\lim_{x\to 6} f(x)$
$\lim_{x\to +\infty} f(x)$	$\lim_{x\to -\infty} f(x)$	

3. Find the following limits:

   a. $\lim_{x\to -1} \frac{x^2-2x-3}{x^2-1}$

   b. $\lim_{x\to -\infty} \frac{x^3+2x-1}{4x^3+x^2}$

   c. $\lim_{x\to 1^-} \frac{x-\sqrt{x}}{x-1}$

   d. $\lim_{x\to -2} \frac{3x+4}{x+2}$

4. Write the definition of the derivative.

5. <u>**Using the definition**</u>, find the derivative of $f(x) = \sqrt{x}$.

6. <u>**Using the definition**</u>, find the derivative of $f(x) = \frac{1}{x}$.

7. List all values of x for which $f(x) = \frac{x^2+1}{xe^x(2x+1)}$ is <u>**not**</u> continuous.

8. List all values of x for which $f(x) = \frac{x(5x-2)}{x^3-3x^2-4x}$ is <u>**not**</u> continuous.

9. Differentiate the following functions:

   a. $y = \frac{x}{\sqrt{x^2-3}}$

   b. $f(x) = (x^2+1)\ln(2x-3)$

   c. $y = \sqrt{e^{-2x}+x^2}$

d. $f(x) = (2x+1)^5(x-2)^6$

10. Find an equation for the tangent line to $f(x) = \frac{4}{x-3}$ at $x = 0$.

11. Find an equation for the tangent line to $f(x) = e^{\sqrt{x}}$ at $x = 1$.

12. Suppose a company has a cost function of $C(x) = \frac{1}{4}x^2 + 3x + 67$ and a demand function of $p(x) = \frac{1}{5}(45 - x)$. x represents the number of units sold.

   a. Find the marginal cost and marginal revenue.
   b. Use marginal cost to estimate the cost of producing the 17th unit. What is the actual cost of producing the 17th unit?
   c. Use marginal revenue to estimate the revenue derived from the sale of the 17th unit. What is the actual revenue obtained from the sale of the 17th unit?

13. Suppose a company makes the following revenue (in thousands of dollars t years after 2005): $R(t) = 0.2t^2 + t + 50$

   a. At what <u>rate</u> is the revenue growing with respect to time in 2008? (Be sure to write the units of your answer.)
   b. At what <u>*percentage* rate</u> is the revenue growing with respect to time in 2008? (Round your percentage to two decimal places.)

14. Use implicit differentiation to find $\frac{dy}{dx}$ given that $2xy + y^2 = x + y$.

15. Find the slope of the tangent line to $3x^2y^2 - 6y = 2$ at $(1,2)$.

16. When the price of a certain commodity is p dollars per unit, consumers demand x hundred units of the commodity, where $75x^2 + 17p^2 = 5300$. How fast is the demand x changing with respect to time when the price is \$7 and is decreasing at the rate of 75 cents per month? (That is, $\frac{dp}{dt} = -0.75$.)

17. When the price of a water bottle is p dollars per unit, producers are willing to supply x hundred bottles, where $2p^2 - 5x^2 = 30$. How fast is the supply x changing with respect to time t when the unit price is \$5 and is increasing at a rate of 50 cents per month? (That is, $\frac{dp}{dt} = 0.5$.)

18. Find the intervals of increase and decrease for the following function: $f(x) = xe^{-2x}$

19. Find the intervals of increase and decrease for the following function: $f(x) = \frac{x^2}{x-3}$

20. Determine where the graph of $f(x) = xe^{-2x}$ is concave upward and concave downward. Also, find the coordinates of any inflection point(s).

21. Determine where the graph of $f(x) = \frac{x-4}{(x+2)^2}$ is concave upward and concave downward. Also, find the coordinates of any inflection point(s).

22. For the function $f(x) = x^3 + 3x^2 - 2$, do the following:

   a. Find the y-intercept of f.
   b. Find $f'(x)$ and $f''(x)$, and determine where each are 0 or do not exist (DNE).
   c. Do a sign analysis on f' and f''.
   d. Find any maxima and minima, as well as any inflection points.
   e. Sketch the graph of f.

23. For the function $f(x) = \frac{x^2+2}{x^2-4}$, do the following:

   a. Find the domain of f.

b. Find the y-intercept of f.
c. Find the horizontal asymptote of f.
d. Find $f'(x)$ and $f''(x)$, and determine where each are 0 or do not exist (DNE).
e. Do a sign analysis on f' and f''.
f. Find any maxima and minima, as well as any inflection points.
g. Sketch the graph of f.

24. Find the absolute maximum and absolute minimum (if any) of…

a. $f(x) = 3x^5 - 5x^3$ on the interval $-2 \le x \le 0$.
b. $f(x) = \frac{x^6}{6} - \frac{4x^5}{5} + x^4$ on the interval $-1 \le x \le 3$.
c. $f(x) = x - \ln x$ on the interval $x > 0$.

25. Use **logarithmic differentiation** to find the derivative of each function:

a. $f(x) = (x+1)^3(6-x)^2\sqrt[3]{2x+1}$
b. $f(x) = \frac{\sqrt{2x+3}}{e^{4x}(x-2)^5}$

26. Evaluate the following integrals:

a. $\int \left(e^{-x} + \sqrt{x^3} - 1 + x^{99} + \frac{5}{x}\right) dx$
b. $\int \frac{x+2}{x^2+4x+2} dx$
c. $\int xe^{x^2} dx$
d. $\int_1^2 \frac{(\ln x)^2}{x} dx$
e. $\int 2xe^{3x} dx$
f. $\int x \ln(2x)\, dx$

27. Find the function $f(x)$ whose tangent line…

a. …has slope $2x + 1$ and whose graph passes through $(0,1)$.
b. …has slope $\frac{2x}{1+3x^2}$ and whose graph passes through $(0,5)$.

28. Find the particular solution of the differential equation satisfying the indicated condition.

a. $\frac{dy}{dx} = \frac{\sqrt{y^2+1}}{xy}$; $y = 2$ when $x = 1$
b. $\frac{dy}{dx} = \frac{2+\sqrt{x}}{y^2}$; through $(0, 3)$

29. Approximate $\int_{-1}^{3} \frac{1}{x+2} dx$…

a. …using the trapezoidal rule with $n = 4$.
b. …using Simpson's rule with $n = 4$.

30. Either evaluate the given improper integral or show that it diverges.

a. $\int_0^{+\infty} (1 + e^{-2x})\, dx$
b. $\int_1^{+\infty} \frac{x^2}{(x^3+1)^2}\, dx$

31. Find the area of the region bounded by…

a. …the lines $y = x$, $y = -x$, and $x = 1$.
b. …the curves $y = x^2 + 2x$ and $y = -x^2 + 4$.

32. Suppose a demand curve (in dollars per unit) is $D(q) = 75e^{-0.03q}$. First, find the price at which 4 units will be demanded (rounded to the nearest cent). Then, compute the consumer surplus at that price.

33. Suppose a supply curve (in dollars per unit) is $S(q) = 0.3q^2 + 30$. First, find the price at which 4 units will be supplied. Then, compute the producer surplus at that price.

34. Find the domain of each function. Then compute the value of the function at the given point.

   a. $f(x,y) = \sqrt{x^2 - y}$ $\quad (3, 5)$
   b. $f(x,y) = \frac{x+y}{(x^2-1)e^{xy}}$ $\quad (0, 1)$

35. Compute f_x and f_y for the following function: $f(x,y) = xe^{xy} + 5x + 2$

36. Compute f_x , f_y , f_{xx} , f_{xy} , f_{yx} , and f_{yy} for the following functions:

   a. $f(x,y) = e^{3x} + 3xy^2 + \ln y$
   b. $f(x,y) = (2x - 3y)^4$

37. The demand functions for a pair of commodities are given. Use partial derivatives to determine whether the commodities are substitute, complementary, or neither.

   a. $D_1 = 400 - 5p_1 + 4p_2;\ \ D_2 = 100 + 2p_1 - 4p_2$
   b. $D_1 = 1500 + \frac{100}{p_1+3} - 20p_2;\ D_2 = 2000 - 100p_1 + \frac{300}{p_2+5}$

38. Find the critical points of the given function and classify each as a relative maximum, a relative minimum, or a saddle point.

   a. $f(x,y) = 12x - x^3 - 4y^2$
   b. $f(x,y) = x^2 + y^2 + x^2y + 4$
   c. $f(x,y) = x^3 + y^3 + 3x^2 - 3y^2$

39. Use the method of Lagrange multipliers to find the maximum and minimum values of…

   a. $f(x,y) = x^2 - y^2 - 2y$ subject to the constraint $x^2 + y^2 = 1$.
   b. $f(x,y) = x^2 + 2y^2 + 2x + 3$ subject to the constraint $x^2 + y^2 = 4$.

40. Evaluate the following double integrals:

   a. $\int_0^4 \int_0^{\sqrt{x}} x^2y\, dydx$
   b. $\int_0^1 \int_0^y y^2e^{xy} dxdy$