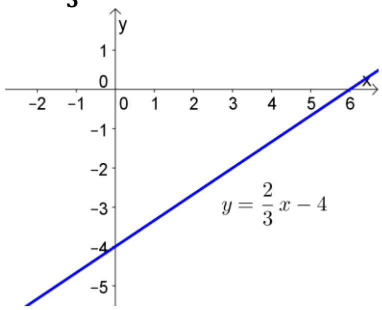
Homework Answer Key

1. Find an equation of the line through $(0, -4)$ with slope $\frac{2}{3}$. Then rewrite the equation in slope-intercept form. Finally, graph the line.

   $\boldsymbol{y + 4 = \frac{2}{3}(x - 0)}$ **(This is point-slope form.)**

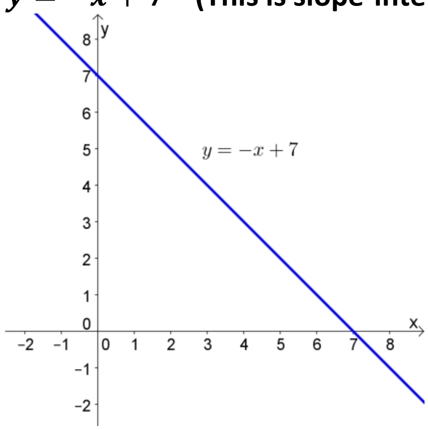
   $\boldsymbol{y = \frac{2}{3}x - 4}$ **(This is slope-intercept form.)**



2. Find an equation of the line through $(2, 5)$ with slope -1. Then rewrite the equation in slope-intercept form. Finally, graph the line.

   $\boldsymbol{y - 5 = -1(x - 2)}$ **(This is point-slope form.)**

   $\boldsymbol{y = -x + 7}$ **(This is slope-intercept form.)**

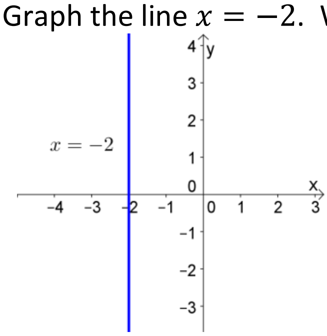


3. Find an equation of the line through $\left(-1, \frac{1}{2}\right)$ and $(0, -\frac{2}{3})$. Then rewrite the equation in slope-intercept form.

   $\boldsymbol{y - \frac{1}{2} = -\frac{7}{6}(x + 1)}$ **(This is point-slope form. You could also write $\boldsymbol{y + \frac{2}{3} = -\frac{7}{6}(x - 0)}$.)**

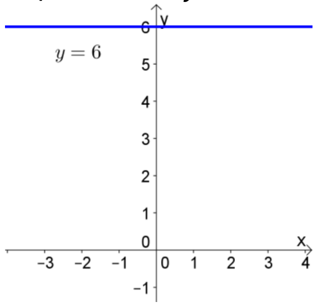
   $\boldsymbol{y = -\frac{7}{6}x - \frac{2}{3}}$ **(This is slope-intercept form.)**

4. Graph the line $x = -2$. What is the slope of the line?



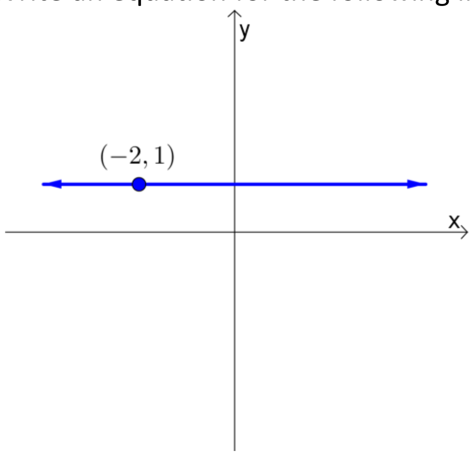
The slope is undefined.

5. Graph the line $y = 6$. What is the slope of the line?



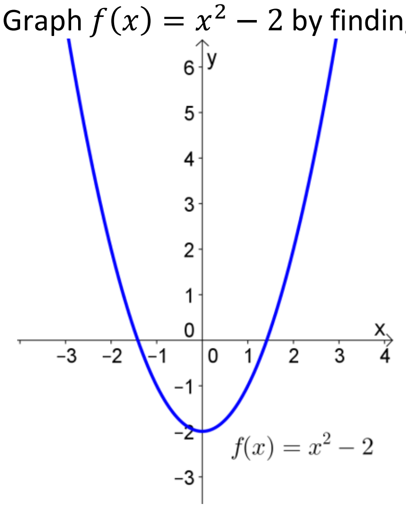
The slope is 0.

6. Write an equation for the following line. What is the slope of the line?



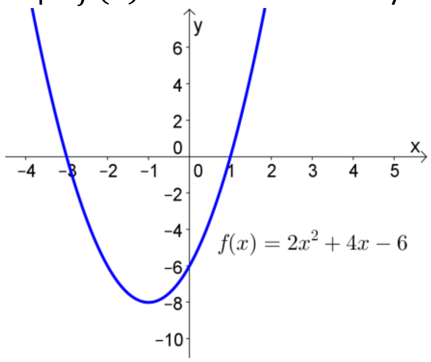
Equation: $\mathbf{y = 1}$
Slope: $\mathbf{0}$

7. Graph $f(x) = x^2 - 2$ by finding and plotting the vertex, x-intercepts, and y-intercept.



Vertex: $\mathbf{(0, -2)}$
$\boldsymbol{x}$**-intercepts:** $\mathbf{(\sqrt{2}, 0)}$ **and** $\mathbf{(-\sqrt{2}, 0)}$
$\boldsymbol{y}$**-intercept:** $\mathbf{(0, -2)}$

8. Graph $f(x) = 2x^2 + 4x - 6$ by finding and plotting the vertex, x-intercepts, and y-intercept.



Vertex: $(\mathbf{-1, -8})$
$\boldsymbol{x}$**-intercepts:** $(\mathbf{-3, 0})$ **and** $(\mathbf{1, 0})$
$\boldsymbol{y}$**-intercept:** $(\mathbf{0, -6})$

9. Suppose $f(x) = \begin{cases} 1 - 3x & \text{if } x \le 4 \\ \frac{2}{x} & \text{if } x > 4 \end{cases}$. Find $f(8)$, $f(0)$, and $f(-1)$.

$\boldsymbol{f(8) = \frac{1}{4}}$ $\boldsymbol{f(0) = 1}$ $\boldsymbol{f(-1) = 4}$

10. Suppose $f(x) = \begin{cases} \sqrt{x + 14} & \text{if } x < -6 \\ 2x^2 + 1 & \text{if} -6 \le x \le -1 \\ \sqrt[3]{x} & \text{if } x > -1 \end{cases}$. Find $f(-10)$, $f(-6)$, $f(-1)$, $f(0)$, and $f(27)$.

$\boldsymbol{f(-10) = 2}$ $\boldsymbol{f(-6) = 73}$ $\boldsymbol{f(-1) = 3}$ $\boldsymbol{f(0) = 0}$ $\boldsymbol{f(27) = 3}$

11. Find $f(x + 1)$ where $f(x) = x^2 - 3x + 2$. Simplify.
$\boldsymbol{f(x + 1) = x^2 - x}$

12. Find $f(x + h)$ where $f(x) = \frac{x}{x+1}$.
$\boldsymbol{f(x + h) = \frac{x+h}{x+h+1}}$

13. Find $\frac{f(x+h)-f(x)}{h}$ where $f(x) = 2x - 3$. Simplify.
$\boldsymbol{\frac{f(x+h)-f(x)}{h} = 2}$

14. Find $\frac{f(x+h)-f(x)}{h}$ where $f(x) = x^2$. Simplify.
$\boldsymbol{\frac{f(x+h)-f(x)}{h} = 2x + h}$

15. Find the domain of $f(x) = \frac{x+1}{x^2-x-2}$.
All real numbers except -1 and 2.

16. Find the domain of $f(x) = \sqrt{6 - 2x}$.
All real numbers such that $\boldsymbol{x \le 3}$.

17. Find the domain of $f(x) = \frac{x-1}{\sqrt{x+2}}$.
All real numbers such that $\boldsymbol{x > -2}$.

18. Factor completely.

a) $3x^4 + 4x^3$
$\boldsymbol{x^3(3x+4)}$

b) $12x^2 + 36x - 48$
$\boldsymbol{12(x-1)(x+4)}$

c) $6x^2 + 6x - 12$
$\boldsymbol{6(x-1)(x+2)}$

d) $x^2 - 2x + 1$
$\boldsymbol{(x-1)^2}$

e) $4x^2 - 9$
$\boldsymbol{(2x-3)(2x+3)}$

f) $x^3 - 4x^2 - 3x + 12$
$\boldsymbol{(x-4)(x^2-3)}$

19. Rewrite the following using rational exponents.

a) $\sqrt{x} = \boldsymbol{x^{\frac{1}{2}}}$ b) $\sqrt[4]{x} = \boldsymbol{x^{\frac{1}{4}}}$ c) $\sqrt[5]{x^2} = \boldsymbol{x^{\frac{2}{5}}}$ d) $\frac{3}{x} = \boldsymbol{3x^{-1}}$

e) $\frac{5}{x^2} = \boldsymbol{5x^{-2}}$ f) $\frac{1}{x^5} = \boldsymbol{x^{-5}}$ g) $\frac{1}{\sqrt{x}} = \boldsymbol{x^{-\frac{1}{2}}}$

20. Rewrite the following using radicals or fractions.

a) $x^{\frac{1}{3}} = \boldsymbol{\sqrt[3]{x}}$ b) $x^{\frac{3}{4}} = \boldsymbol{\sqrt[4]{x^3}}$ c) $x^{\frac{1}{2}} = \boldsymbol{\sqrt{x}}$ d) $4x^{-1} = \boldsymbol{\frac{4}{x}}$

e) $x^{-3} = \boldsymbol{\frac{1}{x^3}}$ f) $x^{-\frac{2}{3}} = \boldsymbol{\frac{1}{\sqrt[3]{x^2}}}$

21. Suppose the demand function for x hundred units of a textbook is:

$D(x) = -0.37x + 47$ (in dollars)

and the cost of producing x hundred textbooks is:

$C(x) = 1.38x^2 + 15.15x + 115.5$ (in hundreds of dollars).

Find the revenue $R(x)$ and profit $P(x)$.

$\boldsymbol{R(x) = -0.37x^2 + 47x}$
$\boldsymbol{P(x) = -1.75x^2 + 31.85x - 115.5}$

22. Suppose that when the price of a water bottle is p dollars per unit, then x thousand units will be purchased by consumers, where $p = -0.01x + 4$. The cost of producing x thousand bottles is $C(x) = 0.005x^2 + x + 120$ thousand dollars.

a) Find the profit function, $P(x)$. (Notice that big P and little p mean two different things. Like passwords, math is case sensitive! ☺)

$\boldsymbol{P(x) = -0.015x^2 + 3x - 120}$

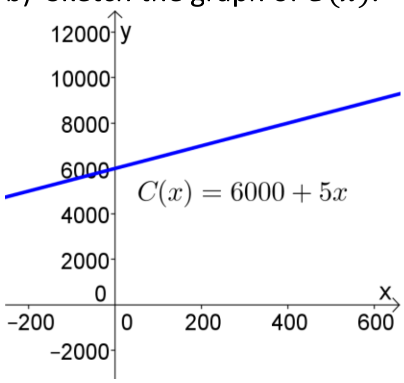
b) Using $P(x)$, determine the level of production x that results in maximum profit.

$\boldsymbol{x = 100 \text{ thousand bottles} = 100,000 \text{ bottles}}$

c) What unit price p corresponds to maximum profit?
$\boldsymbol{p = \$3}$ **per bottle**

23. You start a company that produces mechanical pencils whose lead does not break while writing. Your total cost consists of a fixed overhead of \$6000 plus production costs of \$5 per pencil.
a) Find the total cost function $C(x)$, where x is the number of pencils produced.
$\boldsymbol{C(x) = 6000 + 5x}$

b) Sketch the graph of $C(x)$.



24. You're in the cut-throat iPhone case business. You estimate that you can sell a case for \$2 more than it costs to produce it. Your fixed costs are \$11500.
a) Express total profit $P(x)$ as a function of the level of production x (that is, x represents the number of cases produced).
$\boldsymbol{P(x) = 2x - 11500}$

b) How much profit (or loss) is generated when 5000 cases are produced?
$\boldsymbol{P(5000) = -\$1500}$ **(loss)**

c) What is the level of production at the break-even point? (Hint: Recall that the break-even point occurs when $R(x) = C(x)$. Equivalently, it happens when $P(x) = 0$.)
$\boldsymbol{x = 5750}$ **cases**

25. Suppose the supply and demand functions are $S(x) = 4x + 200$ and $D(x) = -3x + 480$, respectively. Find the value of x_e for which equilibrium occurs and the corresponding equilibrium price p_e.
$\boldsymbol{x_e = 40}$ **and** $\boldsymbol{p_e = 360}$