Constrained Optimization: The Method of Lagrange Multipliers

Q: How can we find the max/min of a two-variable function with a constraint on the variables?
For example, find the max/min values of $f(x,y) = x + y$ subject to the constraint $x^2 + y^2 = 8$.
The **method of Lagrange multipliers** gives us a way to solve such problems. Here are the steps given a function $f(x,y)$ that you want to maximize/minimize, and a constraint $g(x,y) = k$:

1. Find f_x, f_y, g_x, and g_y.
2. Solve the following system of equations (λ is a new variable called the **Lagrange multiplier**):
$$\begin{cases} f_x = \lambda g_x \\ f_y = \lambda g_y \\ g = k \end{cases}$$
3. For each solution (a,b) in step 2, find $f(a,b)$.
4. Largest value from step 3 is max value, and smallest value from step 3 is min value.

Ex 1.
Find the max/min values of $f(x,y) = x + y$ subject to the constraint $x^2 + y^2 = 8$.

Ex 2.

Find the max/min values of $f(x,y) = 8x^2 - 24xy + y^2$ subject to the constraint $8x^2 + y^2 = 1$.

Practice

1. Find the minimum value of $f(x, y) = x^2 + 2y^2 - xy$ subject to the constraint $2x + y = 22$.

Q: A bus driver was heading down a street in Walnut. He went right past a stop sign without stopping, went the wrong way on a one-way street, and then went on the left side of the road past a cop car. The cop did nothing, because he didn't break any traffic laws. Why not?