1. Find all critical points for the function $f(x,y) = xy + \frac{8}{x} + \frac{8}{y}$ and classify each as a relative maximum, a relative minimum, or a saddle point.

$8x^{-1}$ $8y^{-1}$

① $f_x = y - \frac{8}{x^2}$ $f_y = x - \frac{8}{y^2}$

$f_{xx} = \frac{16}{x^3}$ $f_{yy} = \frac{16}{y^3}$

$f_{xy} = 1$

$D(x,y) = f_{xx} f_{yy} - (f_{xy})^2$

$= \left(\frac{16}{x^3}\right)\left(\frac{16}{y^3}\right) - (1)^2$

$= \frac{256}{x^3y^3} - 1$

② Critical points:

$f_x = 0$: and $f_y = 0$:

$y - \frac{8}{x^2} = 0$ $x - \frac{8}{y^2} = 0$

$y = \frac{8}{x^2}$

$x - \frac{8}{\left(\frac{8}{x^2}\right)^2} = 0$

$x - 8 \cdot \frac{x^4}{64} = 0$

$8x - x^4 = 0$

$x(8 - x^3) = 0$

$x = 0$ $8 - x^3 = 0$, $x = 2$

$x = 0$: $y = \frac{8}{0^2}$ ← undefined

$x = 2$: $y = \frac{8}{2^2} = 2$

$(2,2)$ ← critical pt.

③ $D(2,2) = \frac{256}{(2)^3(2)^3} - 1 = 3$ ← positive, so find $f_{xx}(2,2)$

$f_{xx}(2,2) = \frac{16}{(2)^3} = 2$ ← positive, so **relative min at (2,2)**

Q: A sheik announced that a race would decide which of his two sons would inherit all his wealth. The sons were to ride their camels to a certain distant city. The son whose camel reached the city last would be given all of the sheik's wealth. The two sons set out on the journey. After several days of aimless wandering, they met and agreed to seek the advice of a wiseman. After listening to the wiseman's advice, the two sons rode the camels as quickly as possible to the designated city. They did not agree to split the wealth, and their father's decree was to be followed. What was it the wiseman told the two sons?