Partial Derivatives

A _____ is the derivative of a function in two (or more) variables with respect to one variable, while holding all other variables _____.

Ex 1.
Find the partial derivative of $f(x,y) = 3x^2 - 2y$ with respect to x (holding y constant).

Notation:
Suppose $z = f(x,y)$.
$\frac{\partial z}{\partial x}$ or $f_x(x,y)$ means the partial derivative of f with respect to x (holding y constant).
$\frac{\partial z}{\partial y}$ or $f_y(x,y)$ means the partial derivative of f with respect to y (holding x constant).

Ex 2.
Find f_x and f_y if $f(x,y) = 7x^4 + 5xy^3 - \frac{3y}{2x}$.

Ex 3.
Find the partial derivatives $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $z = (x^3 - 2xy + 3y)^6$.

Ex 4.
Find the partial derivatives f_x and f_y if $f(x,y) = ye^{-3xy}$.

There are four _____:

$f_{xx} = (f_x)_x$ $\quad$ $f_{yy} = (f_y)_y$
$f_{xy} = (f_x)_y$ $\quad$ $f_{yx} = (f_y)_x$

Note: f_{xy} and f_{yx} are called _____.

Ex 5.
Compute the four second-order partial derivatives of $f(x,y) = x^2y^3 + 3xy^2 - 2x + y$.

Substitute vs. Complementary Commodities

Sometimes a product can affect another product's demand.

For example, if more cans of Coke are sold, then less bottles of water are likely to be sold. So, Coke and water are said to be _____.

If more hotdogs are sold, then more hotdog buns are likely to be sold. So, hotdogs and hotdog buns are said to be _____.

Ex 6.

Suppose the demand function for hotdogs is given by $D_1(p_1, p_2) = 500 + \frac{10}{p_1+2} - 5p_2$ and the demand for hotdog buns is given by $D_2(p_1, p_2) = 400 - 2p_1 + \frac{7}{p_2+3}$. Here, p_1 is the dollar price for a hotdog, and p_2 is the dollar price for a hotdog bun. Determine whether hotdog and hotdog buns are substitute commodities, complementary commodities, or neither.

Practice

1. Compute all first-order partial derivatives of the given functions.

a) $f(x,y) = 5x^2y + 3xy^3 + 2x - y + 1$

b) $f(x,y) = (x + xy + y)^3$

c) $z = ye^{xy}$

2. Find the second partials (including the mixed partials).

$f(x,y) = 3x^3y^2 - 7xy + y^2$

Q: A man leaves home and, after making three left turns, he ends up back at home, and finds two masked men waiting for him. What is happening?