# Functions of Several Variables

Functions often depend on more than one variable. For example, if a company sells two products (one for \$50 per unit, and one for \$35 per unit), then their revenue function might look like:
$R(x, y) = 50x + 35y$

**Ex 1.**

Suppose $f(x, y) = \frac{4x^3+2x^2-4}{x-y}$. Compute $f(2, -1)$. Also, find the domain of $f$.

**Ex 2.**

Find the domain of $f(x, y) = \sqrt{y - x^2}$.

**Ex 3.**

Compute $f(2, \ln 2, 1)$ if $f(x, y, z) = \frac{e^{xy}}{1+z}$.

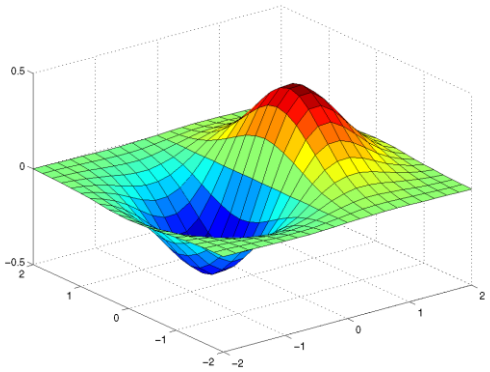
**Notes:**

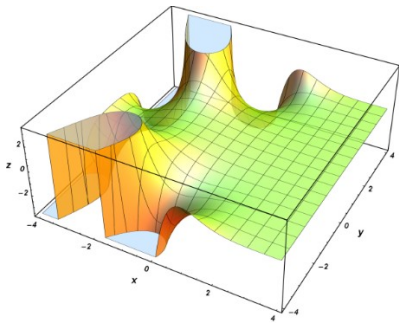
The graph of a function in one variable $f(x)$ is the set of all ordered pairs $(x, y)$ such that $y = f(x)$. (Think: input is $x$, and output is the height $y$.)

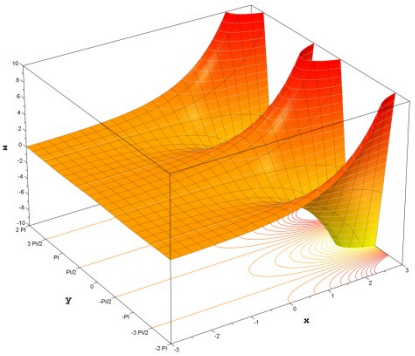
The **graph of a function in two variables** $f(x, y)$ is the set of all triples $(x, y, z)$ such that $z = f(x, y)$. (Think: input is the point $(x, y)$, and output is the height $z$.)

Graphs of functions in one variable, like $f(x)$, are curves.
Graphs of functions in two variables, like $f(x, y)$, are **surfaces**.

Here are some examples of surfaces made from functions:







## Practice

1. Find the domain of $f(x, y) = xye^{xy}$. Find $f(\ln 2, 1)$.

2. Find the domain of $f(x, y) = \ln(y - x^2)$. Find $f(0,1)$.

Q: A man while looking at a photograph said, "Brothers and sisters have I none. That man's father is my father's son." Who was the person in the photograph?