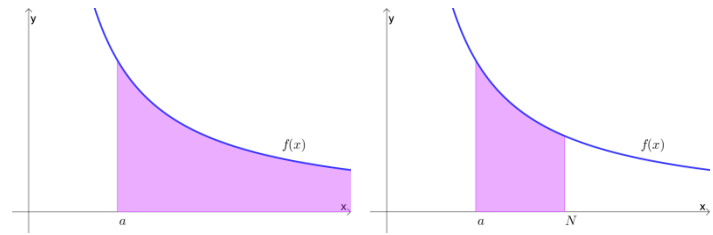


Improper Integrals

Let's define what's called an **improper integral**:

$$\int_a^{+\infty} f(x) dx = \lim_{N \rightarrow +\infty} \int_a^N f(x) dx$$

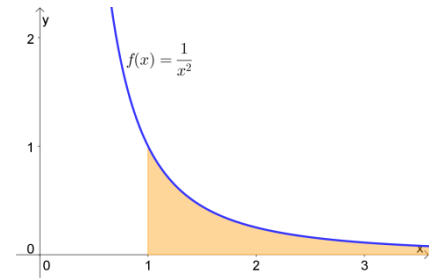
We say that if the above limit exists, then $\int_a^{+\infty} f(x) dx$ **converges**. If not, it **diverges**.



Ex 1.

Either evaluate the given improper integral or show that it diverges.

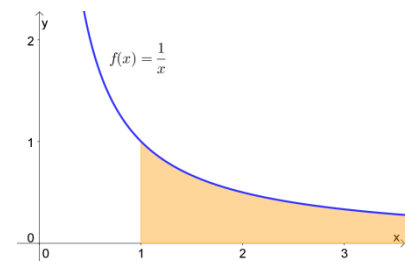
$$\int_1^{+\infty} \frac{1}{x^2} dx$$



Ex 2.

Either evaluate the given improper integral or show that it diverges.

$$\int_1^{+\infty} \frac{1}{x} dx$$



Ex 3.

Either evaluate the given improper integral or show that it diverges.

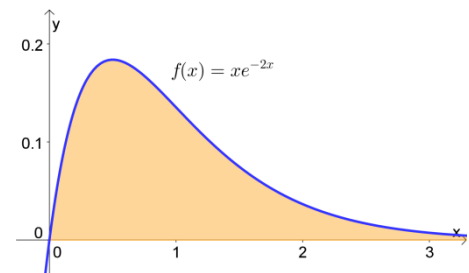
$$\int_0^{+\infty} x e^{-2x} dx$$

Note: In general,

$$\lim_{N \rightarrow +\infty} N^p e^{-kN} = 0$$

ex:

$$\lim_{N \rightarrow +\infty} N^3 e^{-5N} = 0$$



Practice

1. Either evaluate the given improper integral or show that it diverges.

$$\int_1^{+\infty} x^{-3/2} dx$$

Q: What word can you make by adding letters to each side of XYG?
(Hint: add one letter to the left side, and two letters to the right side.)