Numerical Integration

Some functions, like e^{x^2}, do not have elementary antiderivatives. So, what do we do when we need to compute $\int_0^1 e^{x^2}\,dx$? We use numerical integration techniques to approximate the answer.

We have already used rectangles to approximate the area under a curve, but here are two other ways to get good approximations faster.

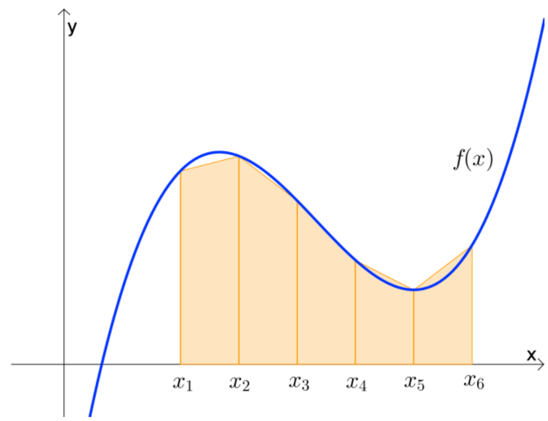
Trapezoids

If we use n trapezoids to approximate the area under a curve, we get the **trapezoidal rule**:

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{2}[f(x_1) + 2f(x_2) + \cdots + 2f(x_n) + f(x_{n+1})$$

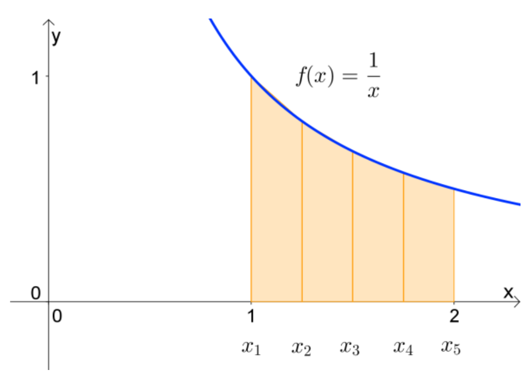
Here, $x_1 = a$, $x_{n+1} = b$, and Δx is the width of each subinterval.

Note: $\Delta x = \frac{b-a}{n}$



Ex 1.

Use the trapezoidal rule with $n = 4$ to approximate $\int_1^2 \frac{1}{x}\,dx$.



Simpson's Rule (Parabolas)

It turns out that using parabolas is another way to get good approximations quickly (see applet).

Simpson's rule is the resulting approximation formula:

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}[f(x_1) + 4f(x_2) + 2f(x_3) + 4f(x_4) + \cdots + 2f(x_{n-1}) + 4f(x_n) + f(x_{n+1})]$$

Note: To use Simpson's Rule, n must be even.

Ex 2.

Use Simpson's rule with $n = 4$ to approximate $\int_1^2 \frac{1}{x}\,dx$.

Practice

1. Use the trapezoidal rule with $n = 4$ to approximate $\int_{-1}^{1}(x^2 - 1)dx$.

Q: Which is correct to say? The yolk of the egg *are* white, or the yolk of the egg *is* white?