# Antidifferentiation: The Indefinite Integral

If you know the rate at which a function is changing,
can you figure out what the function is?

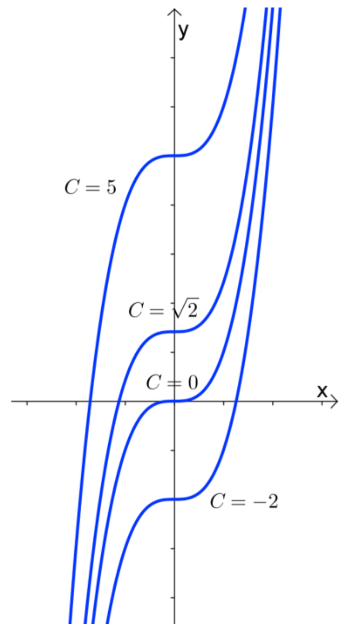
A function $F(x)$ is an ____________________ of $f(x)$ if $F'(x) = f(x)$.

**Ex 1.**
Verify that $F(x) = \frac{2}{3}x^3 - 7x + 5$ is an antiderivative of $f(x) = 2x^2 - 7$.

A function can have many antiderivatives. Take $f(x) = 3x^2$, for example.
All of the following can be antiderivatives:

$x^3, \quad x^3 + 5, \quad x^3 - 2, \quad x^3 + \sqrt{2}$

The family of all antiderivatives of $f(x) = 3x^2$ is ________________.




We use the following notation to describe antiderivatives:

$$\int f(x)\,dx = F(x) + C$$

______________________________ means finding the general antiderivative.

**Common Integration Rules**
(Note: $k$ and $n$ are constants)

$$\int k\,dx = kx + C$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \qquad (\text{for } n \neq -1)$$

$$\int \frac{1}{x}dx = \ln|x| + C$$

$$\int e^{kx}dx = \frac{1}{k}e^{kx} + C$$

**Ex 2.**

Find the following integrals:

$$\int 8\, dx$$

$$\int x^{13}\, dx$$

$$\int \frac{1}{\sqrt{x}}\, dx$$

$$\int e^{-2x}\, dx$$

Just like with derivatives, terms can be integrated separately, and constants can be pulled out.

$$\int f(x) \pm g(x)\, dx = \int f(x)\, dx \pm \int g(x)\, dx$$

$$\int kf(x)\, dx = k \int f(x)\, dx$$

**Ex 3.**

Find the following integral:

$$\int (5x^4 - 6x^3 + x - 3)\, dx$$

**Ex 4.**

Find the function $f(x)$ whose tangent line has slope $3x^2 + 1$ and whose graph passes through $(2,6)$.

**Ex 5.**

A manufacturer has marginal cost $C'(x) = 3x^2 - 60x + 400$ dollars per unit when $x$ units have been made. The total cost of purchasing the first 2 units is $900. What is the total cost of producing the first 5 units?

The equations $\frac{dy}{dx} = 3x^2 + 1$ and $C'(x) = 3x^2 - 60x + 400$ are called ________________________________________, since they are equations with a derivative.

Here are some more examples of differential equations:

ex: $\frac{dy}{dx} = x^2$ $\quad\frac{dA}{dt} = 0.3A$ $\quad\left(\frac{dy}{dx}\right)^2 + 2\frac{dy}{dx} + y = e^x$

**Ex 6.**

Find the general solution of $\frac{dy}{dx} = x^2 + 3x$.

Now find the particular solution of $\frac{dy}{dx} = x^2 + 3x$ that satisfies $y = 2$ when $x = 1$.

Note: A differential equation along with an initial condition together make an __________________________________________. (see Ex 5 and Ex 6)

**Ex 7.**

Find the general solution of the differential equation $\frac{dy}{dx} = \frac{2x}{y^2}$.

In general, a differential equation that can be written in the form $g(y)dy = h(x)dx$ is called a _________________ equation.

## Practice

1. Find the following integrals:

   a) $\int 3x^5 dx$

   b) $\int (\sqrt[3]{x} - 4)dx$

   c) $\int \left(\frac{1}{x} + e^{7x}\right) dx$

2. Find the function $f(x)$ whose tangent line has slope $3x + 2$ and whose graph passes through $(3,0)$.

3. Find the *particular* solution of the differential equation $\frac{dy}{dx} = 4x^3y^2$, given the condition that $y = 2$ when $x = 1$.

Q: What is the word that everybody always says wrong?