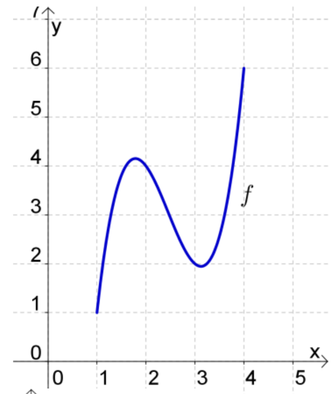
Optimization; Elasticity of Demand

We know how to find *relative* minima and maxima, but how do we find *the* minimum and maximum of a function?



Suppose we have a function f that's defined on an interval I (where c is in I).

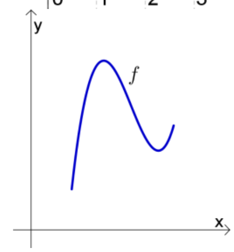
$f(c)$ is the ________________________ of f on I if $f(c) \geq f(x)$ for all x in I.

$f(c)$ is the ________________________ of f on I if $f(c) \leq f(x)$ for all x in I.

Q: Are we guaranteed to have an absolute max and min?

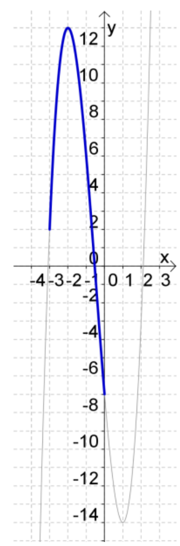
A: The **Extreme Value Property** says "yes" under certain conditions:

If $f(x)$ is continuous on the closed interval $a \leq x \leq b$, then it will have an absolute max and min on the interval at either a, b, or a critical number c (where $a < c < b$).



Ex 1.

Find the absolute maximum and absolute minimum of the function $f(x) = 2x^3 + 3x^2 - 12x - 7$ on the interval $-3 \leq x \leq 0$.



If your interval doesn't have 2 endpoints, and only has one critical number, you can use…

The Second Derivative Test for Absolute Extrema

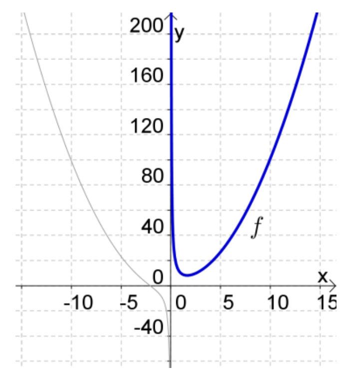
Suppose $f(x)$ is continuous on an interval I where $x = c$ is <u>the only</u> critical number, and $f'(c) = 0$.

Then…if $f''(c) > 0$, the absolute min of f on I is $f(c)$.

…if $f''(c) < 0$, the absolute max of f on I is $f(c)$.

Ex 2.

Find the absolute maximum and absolute minimum (if any) of the function $f(x) = x^2 + \frac{16}{x}$ on the interval $x > 0$.



Ex 3.

A manufacturer estimates that when q thousand units of a particular commodity are produced each month, the total cost will be $C(q) = 0.4q^2 + 3q + 40$ thousand dollars, and all q thousand units can be sold at a price of $p(q) = 22.2 - 1.2q$ dollars per unit. Determine the level of production that results in a maximum profit. What is the maximum profit?

At what level of production is the average cost per unit $A(q) = \frac{C(q)}{q}$ minimized?

Practice

1. Find the absolute maximum and absolute minimum (if any) of the function $f(x) = x^5 - 5x^4 + 1$ on the interval $0 \leq x \leq 5$.

2. Find the absolute maximum and absolute minimum (if any) of the function $f(x) = x + \frac{1}{x-1}$ on the interval $x > 1$.

Q: What comes once in a minute, twice in a moment, but never in a thousand years?