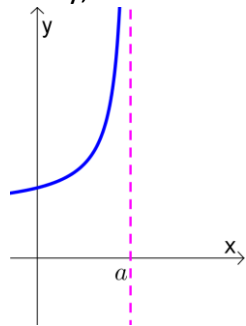
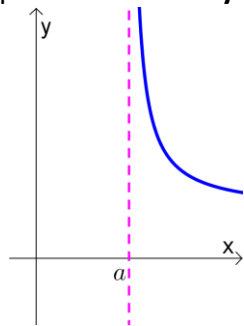


Curve Sketching

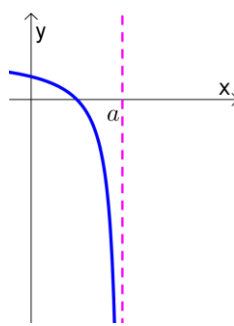
Visually, here are the four types of **vertical asymptotes**:



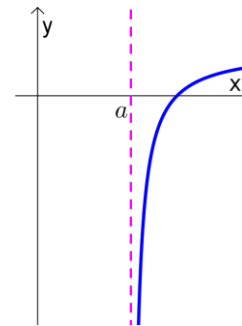
When $x \rightarrow a^-, f(x) \rightarrow \infty$



When $x \rightarrow a^+, f(x) \rightarrow \infty$



When $x \rightarrow a^-, f(x) \rightarrow -\infty$



When $x \rightarrow a^+, f(x) \rightarrow -\infty$

$f(x)$ has a _____ at $x = c$ if:

$$\lim_{x \rightarrow c^-} f(x) = +\infty \quad (\text{or } -\infty)$$

or

$$\lim_{x \rightarrow c^+} f(x) = +\infty \quad (\text{or } -\infty)$$

For a rational function $\frac{p(x)}{q(x)}$, we get a vertical asymptote at $x = c$ if $q(c) = 0$ and $p(c) \neq 0$.

(That is, if bottom = 0 and top \neq 0.)

Ex 1.

Find all vertical asymptotes of the graph of $f(x) = \frac{x^2 - 9}{x^2 + 3x}$

$f(x)$ has a _____ $y = b$ if:

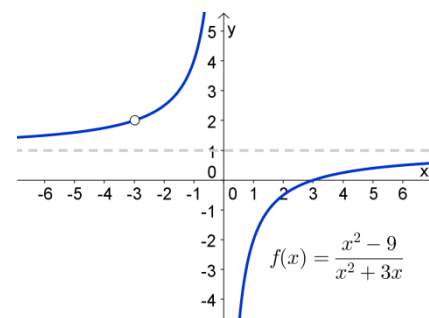
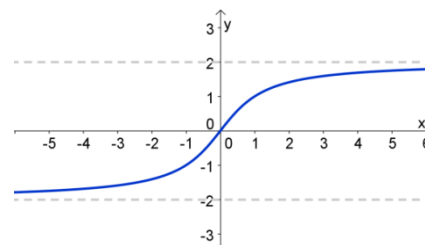
$$\lim_{x \rightarrow +\infty} f(x) = b$$

or

$$\lim_{x \rightarrow -\infty} f(x) = b$$

Ex 2.

Find all horizontal asymptotes of the graph of $f(x) = \frac{x^2 - 9}{x^2 + 3x}$

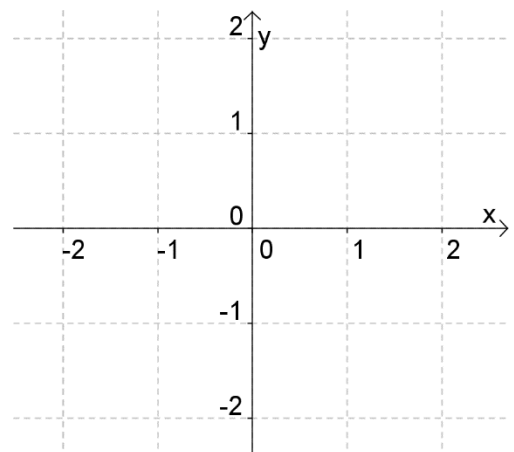


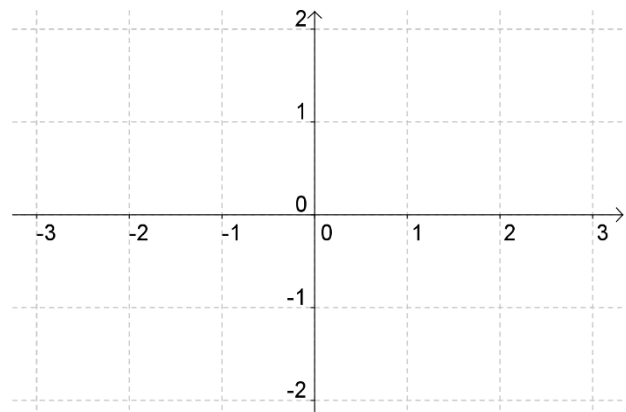
How to sketch a curve (using the techniques we know so far):

1. Find **domain**.
2. Find/plot **intercepts**. (if possible)
3. Find/draw **asymptotes**.
4. Find f' and f'' , and determine when each are **0** or **DNE**.
5. Do a **sign analysis** on f' and f'' .
6. Find/plot **maxima/minima**, and **inflection points**.
7. Sketch curve.

Ex 3.

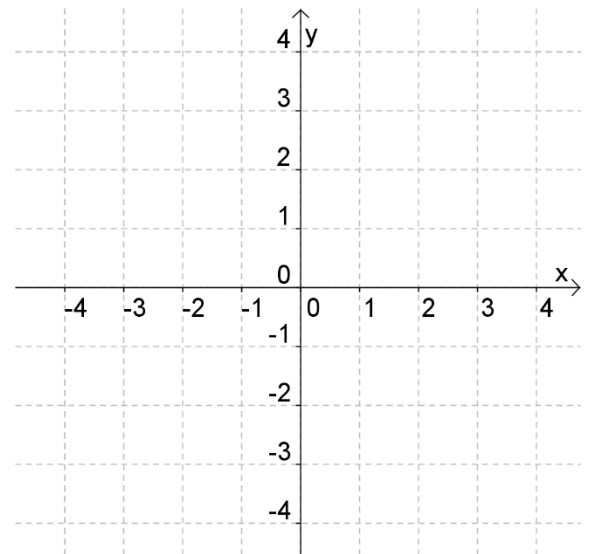
Sketch the graph of $f(x) = 3x^4 + 4x^3$



Ex 4.Sketch the graph of $f(x) = \frac{x}{(x+1)^2}$ 

Practice

1. Find the domain, intercepts, asymptotes, relative extrema, and inflection points, and sketch the graph of $f(x) = \frac{2x^2}{x^2-1}$



Q: What belongs to you but others use it more than you do?