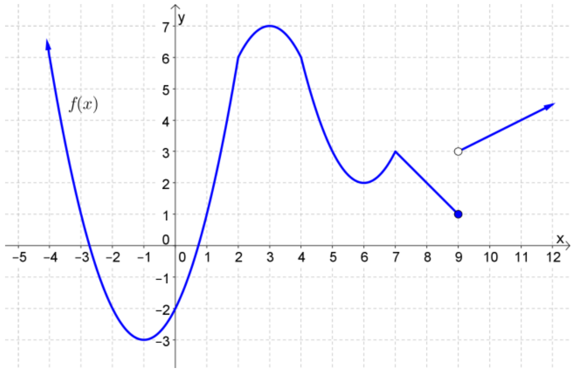
Increasing and Decreasing Functions; Relative Extrema



Let $f(x)$ be defined on the interval $a < x < b$. Suppose x_1 and x_2 are in that interval.

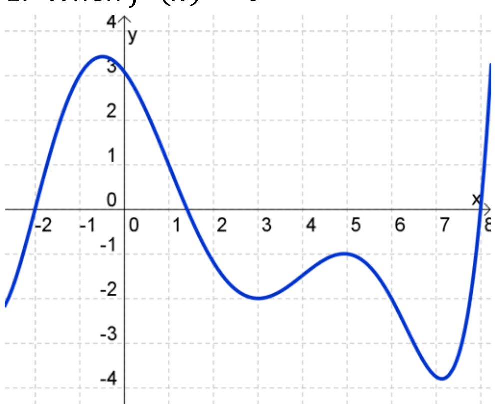
f is ____________________ on that interval if $f(x_2) > f(x_1)$ when $x_2 > x_1$

f is ____________________ on that interval if $f(x_2) < f(x_1)$ when $x_2 > x_1$

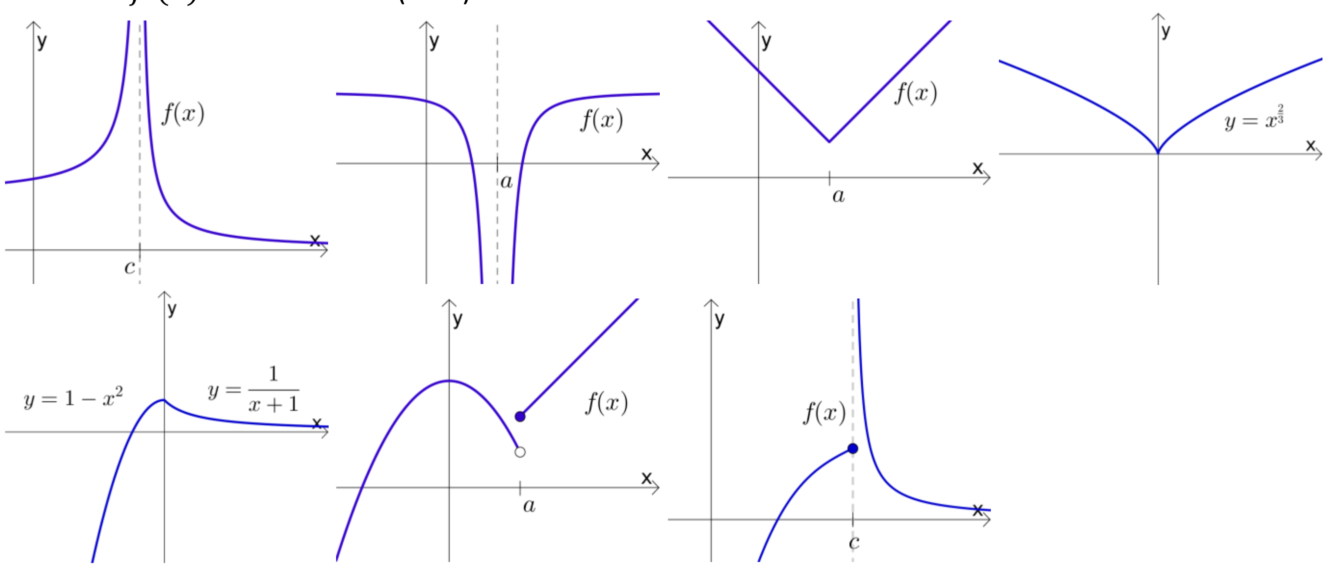
Recall: f is increasing where $f'(x) > 0$, and decreasing where $f'(x) < 0$.

Where might $f(x)$ change from increasing to decreasing, or decreasing to increasing?

1. When $f'(x) = 0$

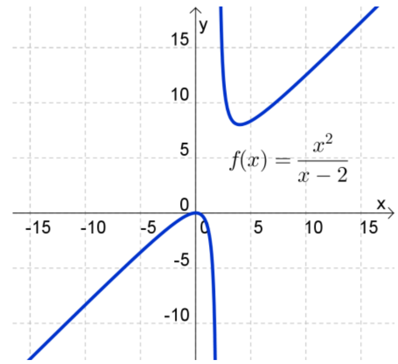


2. When $f'(x)$ does not exist (DNE)

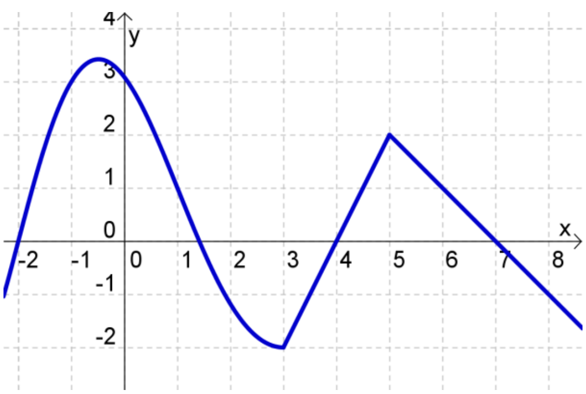


Ex 1.

Find the intervals of increase and decrease for $f(x) = \frac{x^2}{x-2}$



Relative Extrema



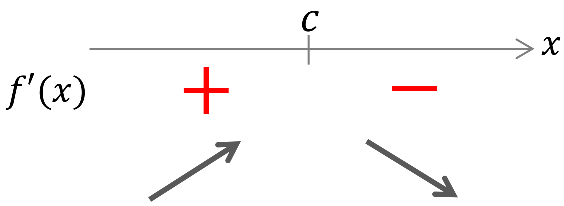
If $f(c)$ is defined, and either $f'(c) = 0$ or $f'(c)$ DNE, then $x = c$ is called a ________________________.

Also, $(c, f(c))$ is called a _____________________________.

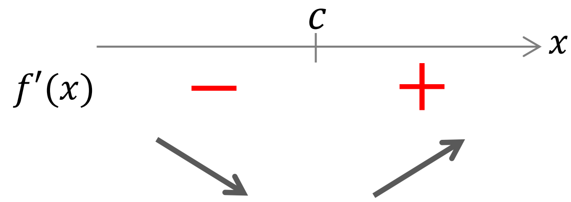
Critical numbers give us x-values where relative extrema <u>might</u> occur.

First Derivative Test for Relative Extrema

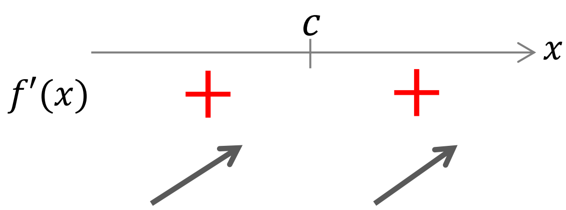
Let c be a critical number.



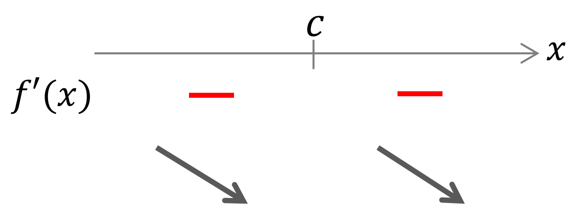
Relative
max at
$(c, f(c))$



Relative
min at
$(c, f(c))$



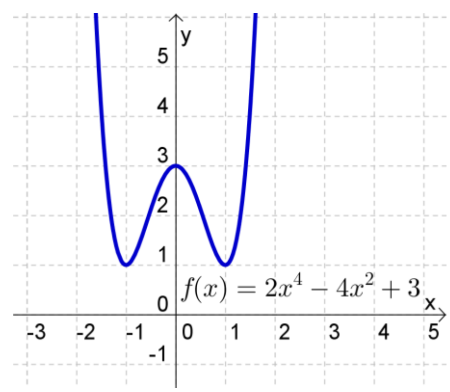
Not a relative
extremum



Not a relative
extremum

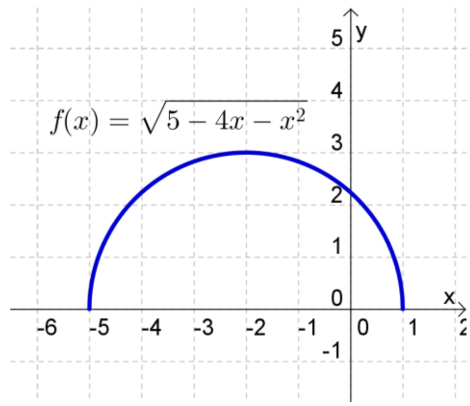
Ex 2.

Find all critical numbers for $f(x) = 2x^4 - 4x^2 + 3$, and classify each critical point as a relative maximum, relative minimum, or neither.



Ex 3.

Find all critical numbers for $f(x) = \sqrt{5 - 4x - x^2}$, and classify each critical point as a relative maximum, relative minimum, or neither.



Practice

1. Find the intervals of increase and decrease for $f(x) = 2x^3 + 3x^2 - 12x - 7$

2. Find all critical numbers for $f(x) = 3x^4 - \frac{3}{2}x^2 + 1$, and classify each critical point as a relative maximum, relative minimum, or neither.

Q: What goes up and never comes down?