Implicit Differentiation and Related Rates

The following equations are in _______________ form:

$y = 2x^2 + x - 3$ $\qquad y = \frac{x^2+1}{3x-5}$ $\qquad y = \sqrt{x-3}$

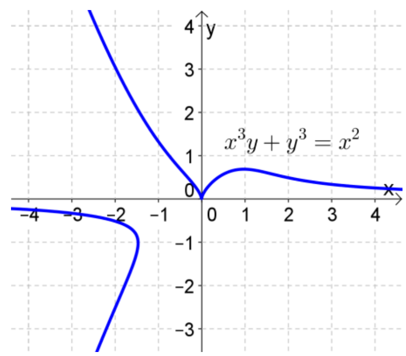
Sometimes equations will be in _______________ form, for example:

$x^3y + y^3 = x^2$ $\qquad x^2 + y^2 = 25$ $\qquad x^2 - y^2 = 2x + 4y$

How do we find $\frac{dy}{dx}$ if an equation is in implicit form? Note that x and y are related by the equation. If x changes, then y changes. So you can think of y as a "function" of x, and differentiate both sides of the equation as if they were functions of x.

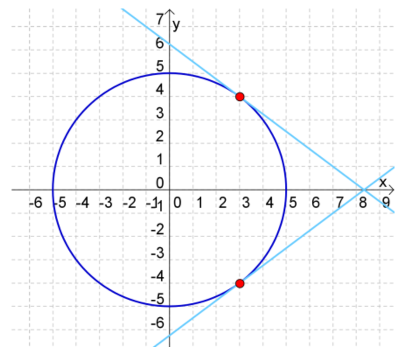
Ex 1.

Find $\frac{dy}{dx}$ given that $x^3y + y^3 = x^2$.



Ex 2.

Find the slope of the tangent line to $x^2 + y^2 = 25$ at $(3, 4)$ and $(3, -4)$.



Ex 3.

When the price of a water bottle is p dollars per unit, consumers demand x hundred bottles, where $3x^2 + 5p^2 = 1000$. How fast is the demand x changing with respect to time t when the unit price is \$8 and is decreasing at a rate of \$1 per month? (That is, $\frac{dp}{dt} = -1$.)

Practice

1. Find $\frac{dy}{dx}$ given that $x^2y^3 + 2y = 3x$.

2. When the price of a water bottle is p dollars per unit, producers are willing to supply x hundred bottles, where $3p^2 - 2x^2 = 60$. How fast is the supply x changing with respect to time t when the unit price is $6 and is increasing at a rate of 80 cents per month? (That is, $\frac{dp}{dt} = 0.8$.)

3. Consider the relation $x^2 - y^2 = 2x + 4y$.

a) Find all points on $x^2 - y^2 = 2x + 4y$ where the tangent line is horizontal. (Hint: Find $\frac{dy}{dx}$ and then figure out where $\frac{dy}{dx} = 0$.)

b) Now find all points on $x^2 - y^2 = 2x + 4y$ where the tangent line is vertical. (Hint: When would $\frac{dy}{dx}$ be undefined?)

Q: What word starts with "e" and has only one letter in it?