1. Suppose a company has a cost function of $C(x) = \frac{1}{4}x^2 + 3x + 67$ dollars and a demand function of $p(x) = \frac{1}{5}(45 - x)$ dollars per unit. x represents the number of units sold.

a) Find the marginal cost and marginal revenue.

$\boxed{C'(x) = \frac{1}{2}x + 3}$ ← marginal cost

$R(x) = x\,p(x) = x \cdot \frac{1}{5}(45 - x) = 9x - \frac{1}{5}x^2$

$\boxed{R'(x) = 9 - \frac{2}{5}x}$ ← marginal revenue

b) Use marginal cost to estimate the cost of producing the 21st unit. What is the actual cost of producing the 21st unit?

$C'(20) = \frac{1}{2}(20) + 3 = 13 \rightarrow$ $\boxed{\$13 \text{ per unit}}$ (estimate)

$C(20) = \frac{1}{4}(20)^2 + 3(20) + 67 = 227$

$C(21) = \frac{1}{4}(21)^2 + 3(21) + 67 = 240.25$

$C(21) - C(20) = 240.25 - 227 = 13.25 \rightarrow$ $\boxed{\$13.25 \text{ per unit}}$ (actual)

c) Use marginal revenue to estimate the revenue derived from the sale of the 21st unit. What is the actual revenue obtained from the sale of the 21st unit?

$R'(20) = 9 - \frac{2}{5}(20) = 1 \rightarrow$ $\boxed{\$1 \text{ per unit}}$ (estimate)

$R(20) = 9(20) - \frac{1}{5}(20)^2 = 100$

$R(21) = 9(21) - \frac{1}{5}(21)^2 = 100.8$

$R(21) - R(20) = 100.8 - 100 = 0.8 \rightarrow$ $\boxed{\$0.80 \text{ per unit}}$ (actual)

Q: What starts with "P" and ends with "E" and has more than 1000 letters?