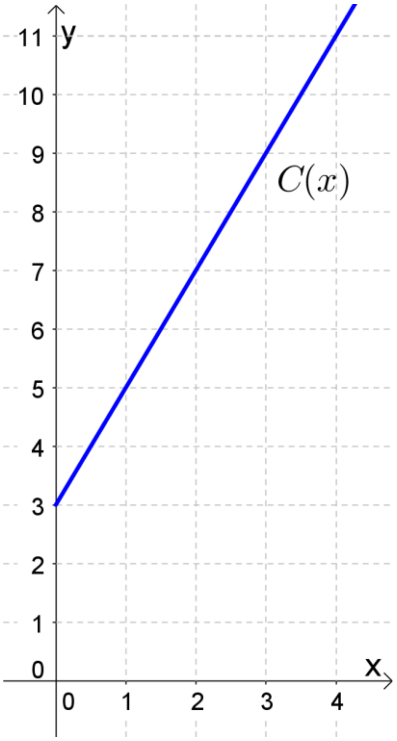
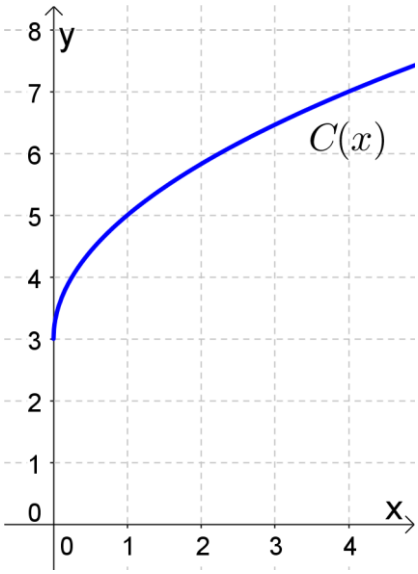
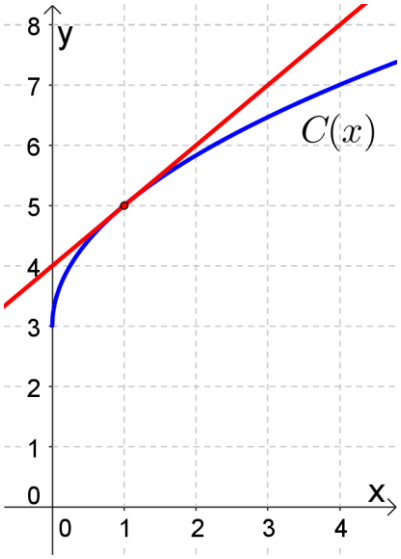
Marginal Analysis

Marginal analysis looks at the change in cost, revenue, etc. that results from a 1-unit increase in production. Let's look at how marginal analysis applies to the following cost functions:







When economists get theoretical, they just use derivatives to approximate marginal cost, marginal revenue, etc.

$R'(x)$ is called _____.

$C'(x)$ is called _____.

$P'(x)$ is called _____.

Ex 1.

Pack My Back, Inc. estimates that when x backpacks are produced, the total cost will be $C(x) = \frac{1}{8}x^2 + 3x + 98$ dollars. All x backpacks will be sold when the price is $p(x) = \frac{1}{3}(75 - x)$ dollars per backpack.

a) Find the marginal cost and the marginal revenue.

b) Use marginal cost to estimate the cost of producing the 37th unit. What is the actual cost of producing the 37th unit?

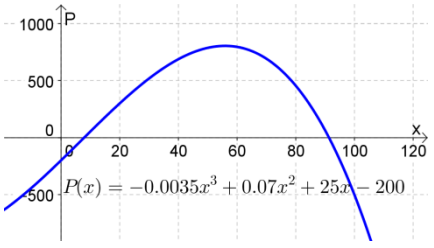
Ex 2.

A business manager named Joe estimates that when x hundred phones are produced, the total profit will be P thousand dollars, where $P(x) = -0.0035x^3 + 0.07x^2 + 25x - 200$.

a) What is the marginal profit function?

b) Based on a current level of production $x = 10$, should Joe increase or decrease production?

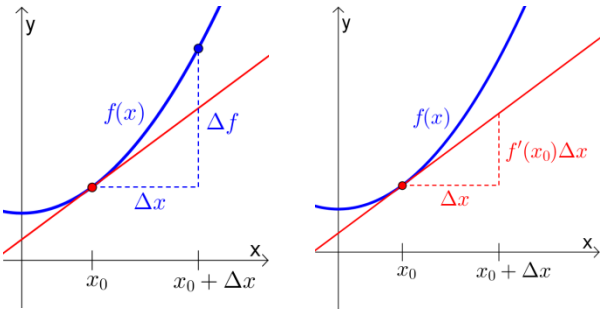
c) What decision should Joe make if $x = 50$? How about if $x = 80$?



Generally, we can use derivatives to estimate the change in a function (Δf) determined by a change in its input (Δx). Here's how:

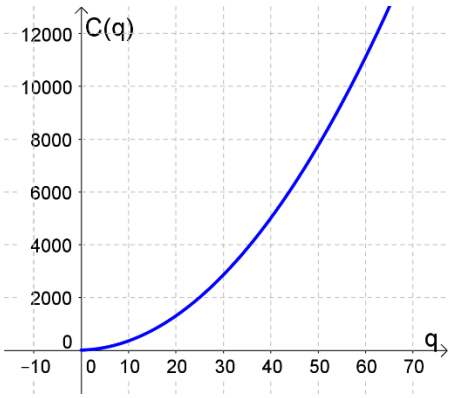
$\frac{\Delta f}{\Delta x} \approx f'(x_0)$ so $\boldsymbol{\Delta f \approx f'(x_0)\Delta x}$

For example, $\Delta P \approx P'(x_0)\Delta x$. Note that if $\Delta x = 1$, then this becomes $\Delta P \approx P'(x_0)$, which is just marginal profit.



Ex 3.

The total cost of manufacturing q hundred units of a certain commodity is C thousand dollars, where $C(q) = 3q^2 + 5q + 10$. If the current level of production is 4000 units, estimate how the total cost will change if 4050 units are produced.



Practice

1. Suppose a company has a cost function of $C(x) = \frac{1}{4}x^2 + 3x + 67$ dollars and a demand function of $p(x) = \frac{1}{5}(45 - x)$ dollars per unit. x represents the number of units sold.

   a) Find the marginal cost and marginal revenue.

b) Use marginal cost to estimate the cost of producing the 21^{st} unit. What is the actual cost of producing the 21^{st} unit?

c) Use marginal revenue to estimate the revenue derived from the sale of the 21^{st} unit. What is the actual revenue obtained from the sale of the 21^{st} unit?

Q: What starts with "P" and ends with "E" and has more than 1000 letters?