Techniques of Differentiation

Here are some shortcuts to computing the derivative.

Constant Rule: $\frac{d}{dx}(c) = 0$ **(here, c is a constant)**

ex: $\frac{d}{dx}(-15) = 0$

Power Rule: $\frac{d}{dx}(x^n) = nx^{n-1}$

ex: $\frac{d}{dx}(x^3) =$

ex: $\frac{d}{dx}\left(\sqrt{x}\right) =$

Constant Multiple Rule: $\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$

ex: $\frac{d}{dx}(3x^4) =$

ex: $\frac{d}{dx}\left(-\frac{7}{\sqrt{x}}\right) =$

Sum Rule: $\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)]$

ex: $\frac{d}{dx}(x^{-2} + 7) =$

Relative and Percentage Rates of Change

If you earn $50,000 and get a raise of $2,000, then the **relative rate of change** in your salary is:

$\frac{\$2,000}{\$50,000} = 0.04$

This can be interpreted as a **percentage rate of change** of 4%.

In general, for a function $f(x)$ that is constantly changing, you can calculate the instantaneous relative rate of change by computing:

$\frac{f'(x)}{f(x)}$

ex: Suppose a company makes the following profit (in thousands of dollars t years after 2005):
$P(t) = 0.2t^2 + 5t + 10$
At what rate is the profit growing with respect to time in 2009?

At what *percentage* rate is the profit growing with respect to time in 2009?

Practice

1. Find each of the following.

a) $\frac{d}{dx}(x^2 + x + 1)$

b) $\frac{d}{dx}\left(\sqrt[3]{x^2}\right)$

c) $\frac{d}{dx}\left(\frac{1}{x^3} - 2\right)$

d) $\frac{d}{dx}(-1000)$

e) $\frac{d}{dx}(x^{1.4})$

f) $\frac{d}{dx}(2x^{27})$

g) $\frac{d}{dx}\left(-\frac{3}{x^2}\right)$

2. Suppose a company makes the following revenue (in millions of dollars t years after 2002):
$R(t) = 0.4\sqrt{t} + 2$
a) At what rate is the revenue growing with respect to time in 2007?

b) At what percentage rate is the revenue growing with respect to time in 2007?

Q: What goes around the world but stays in a corner?