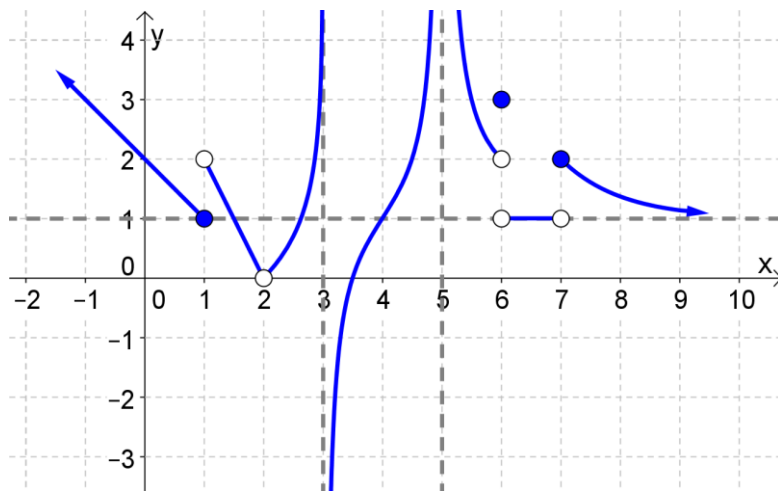


1. Suppose you have the following piecewise-defined function,  $f(x)$ .



a) Find the following.

$$\lim_{x \rightarrow 1^-} f(x) = \mathbf{1}$$

$$\lim_{x \rightarrow 1^+} f(x) = \mathbf{2}$$

$$\lim_{x \rightarrow 1} f(x) \text{ **DNE**}$$

$$\lim_{x \rightarrow 2^-} f(x) = \mathbf{0}$$

$$\lim_{x \rightarrow 2^+} f(x) = \mathbf{0}$$

$$\lim_{x \rightarrow 2} f(x) = \mathbf{0}$$

$$\lim_{x \rightarrow 3^-} f(x) = \mathbf{+\infty}$$

$$\lim_{x \rightarrow 3^+} f(x) = \mathbf{-\infty}$$

$$\lim_{x \rightarrow 3} f(x) \text{ **DNE**}$$

$$\lim_{x \rightarrow 4^-} f(x) = \mathbf{1}$$

$$\lim_{x \rightarrow 4^+} f(x) = \mathbf{1}$$

$$\lim_{x \rightarrow 4} f(x) = \mathbf{1}$$

$$\lim_{x \rightarrow 5^-} f(x) = \mathbf{+\infty}$$

$$\lim_{x \rightarrow 5^+} f(x) = \mathbf{+\infty}$$

$$\lim_{x \rightarrow 5} f(x) = \mathbf{+\infty}$$

$$\lim_{x \rightarrow 6^-} f(x) = \mathbf{2}$$

$$\lim_{x \rightarrow 6^+} f(x) = \mathbf{1}$$

$$\lim_{x \rightarrow 6} f(x) \text{ **DNE**}$$

$$\lim_{x \rightarrow 7^-} f(x) = \mathbf{1}$$

$$\lim_{x \rightarrow 7^+} f(x) = \mathbf{2}$$

$$\lim_{x \rightarrow 7} f(x) \text{ **DNE**}$$

$$\lim_{x \rightarrow +\infty} f(x) = \mathbf{1}$$

$$\lim_{x \rightarrow -\infty} f(x) = \mathbf{+\infty}$$

$$f(1) = \mathbf{1}$$

$$f(2) \text{ **undefined**}$$

$$f(3) \text{ **undefined**}$$

$$f(4) = \mathbf{1}$$

$$f(5) \text{ **undefined**}$$

$$f(6) = \mathbf{3}$$

$$f(7) = \mathbf{2}$$

b) List all values of  $x$  for which  $f(x)$  is not continuous.

$$\mathbf{x = 1, 2, 3, 5, 6, 7}$$

2. Find each of the following limits.

a)  $\lim_{x \rightarrow 3} (x^2 - 2x + 1) = \mathbf{4}$

b)  $\lim_{x \rightarrow -3} \frac{5-2x}{x-3} = \mathbf{-\frac{11}{6}}$

c)  $\lim_{x \rightarrow 2} \frac{x^2-4}{x-2} = \mathbf{4}$

d)  $\lim_{x \rightarrow \frac{1}{2}} \frac{2x^2+5x-3}{2x-1} = \mathbf{3\frac{1}{2}}$  (or  $\frac{7}{2}$ )

$$e) \lim_{x \rightarrow 1} \frac{x^2 + 2x - 3}{2x^2 + x - 3} = \frac{4}{5}$$

$$f) \lim_{x \rightarrow 1^-} \frac{x - \sqrt{x}}{x - 1} = \frac{1}{2} \quad (\text{Hint: multiply top and bottom by } x + \sqrt{x}, \text{ then factor the top})$$

$$g) \lim_{x \rightarrow -2^+} \frac{x+3}{x+2} = +\infty$$

$$h) \lim_{x \rightarrow 5} \frac{2x-1}{x-5} \text{ DNE}$$

$$i) \lim_{x \rightarrow -2} \frac{x+1}{x+2} \text{ DNE}$$

$$j) \lim_{x \rightarrow +\infty} \frac{3x^3 + 4x - 1}{2x^4 - 7x + 8} = 0$$

$$k) \lim_{x \rightarrow -\infty} \frac{3x^2 - 2x + 5}{4x^2 + x + 7} = \frac{3}{4}$$

$$l) \lim_{x \rightarrow -\infty} \frac{1 - 2x^3}{x + 1} = -\infty$$

3. Find the following limits given that  $f(x) = \begin{cases} 2x^2 & \text{if } x \leq 3 \\ x - 1 & \text{if } x > 3 \end{cases}$

$$a) \lim_{x \rightarrow 3^-} f(x) = \mathbf{18}$$

$$b) \lim_{x \rightarrow 3^+} f(x) = \mathbf{2}$$

$$c) \lim_{x \rightarrow 3} f(x) \text{ DNE}$$

d) Is  $f(x)$  continuous at  $x = 3$ ? Why or why not? **No, since  $\lim_{x \rightarrow 3} f(x)$  DNE.**

4. Is  $f(x) = \frac{x-2}{x+5}$  continuous at  $x = -5$ ? Why or why not? **No, since  $f(-5)$  is undefined.**

5. Is  $f(x) = x^3 - 4x + 9$  continuous at  $x = 2$ ? Why or why not? **Yes, since polynomials are continuous for all  $x$ .**

6. Is  $f(x) = \begin{cases} 2 - 3x & \text{if } x < -1 \\ x^2 - x + 3 & \text{if } x \geq -1 \end{cases}$  continuous at  $x = -1$ ? Why or why not? (Be sure to use the definition of continuous. That is, show  $\lim_{x \rightarrow -1} f(x) = f(-1)$ .) **Yes. (To show your work, find**

**$\lim_{x \rightarrow -1^-} f(x)$ ,  $\lim_{x \rightarrow -1^+} f(x)$ ,  $\lim_{x \rightarrow -1} f(x)$ , and  $f(-1)$ . Then just state that since  $\lim_{x \rightarrow -1} f(x) = f(-1)$ ,  $f(x)$  is continuous at  $x = -1$ .)**

7. List all values of  $x$  for which  $f(x) = \frac{x+2}{x^3-4x}$  is not continuous.  $x = \mathbf{0, 2, -2}$

8. List all values of  $x$  for which  $f(x) = \begin{cases} x^2 - 5 & \text{if } x < 0 \\ \frac{2}{3}x + 1 & \text{if } x \geq 0 \end{cases}$  is not continuous.  $x = \mathbf{0}$  (Since

**$\lim_{x \rightarrow 0^-} f(x) \neq \lim_{x \rightarrow 0^+} f(x)$  and so  $\lim_{x \rightarrow 0} f(x)$  DNE.)**

9. You start a company that sells a business calculus textbook.

a) Suppose your total costs one month are \$20000, and you produce 400 textbooks that month.

Find the average cost per textbook for that month. Be sure to write the units for your answer.

**\$50 per textbook**

b) Now suppose that  $C(x) = 25x + 10000$  represents your total costs (in dollars), where  $x$  is the number of textbooks produced. Find the average cost per textbook (call it  $A(x)$ ).

$$A(x) = 25 + \frac{10000}{x}$$

c) Find  $\lim_{x \rightarrow +\infty} A(x)$ . Be sure to write the units for your answer.  **$\lim_{x \rightarrow +\infty} A(x) = 25$ ,**

**\$25 per textbook**

d) What does your answer to part (c) mean? **The more textbooks produced, the closer the “per textbook” cost gets to \$25.**