Quiz #4 – Take-home (15 points total) Name: Solutions

Math 140, Prof. Beydler Due date: Wednesday, December 7, 2016

Directions: Show all work. You may use your notes and book. It's okay to get help, just be sure you're not copying someone else's work. Please box your answers.

1. (1 point) Find the domain of each function. Then compute the value of the function at the given point.

$f(x,y) = \frac{e^{xy}}{\sqrt{x-2y}}$ $(0,-2)$

Domain: $x-2y>0 \rightarrow y<\frac{1}{2}x$

$\boxed{\text{All } (x,y) \text{ such that } y<\frac{1}{2}x}$

$f(0,-2) = \frac{e^{(0)(-2)}}{\sqrt{0-2(-2)}} = \boxed{\frac{1}{2}}$

2. (2 points) Compute f_{yx} for $f(x,y) = x^2y^3 + 3xy^2 - 2x + y$.

$f_y = 3x^2y^2 + 6xy + 1$

$\boxed{f_{yx} = 6xy^2 + 6y}$

3. (5 points) Use the method of Langrage multipliers to find the minimum value of $f(x,y) = x^2 + xy + y^2$ subject to the constraint $2x - y = \frac{70}{5}$.

1 $f_x = 2x+y$ $g_x = 2$

$f_y = x+2y$ $g_y = -1$

2 $\begin{cases} 2x+y = 2\lambda \\ x+2y = -\lambda \\ 2x-y = \frac{70}{5} \end{cases}$ $\xrightarrow{\text{Divide}}$ $\frac{2x+y}{x+2y} = -2 \rightarrow 2x+y = -2x-4y \rightarrow 5y = -4x$

$y = -\frac{4}{5}x$

$2x - \left(-\frac{4}{5}x\right) = \frac{70}{5}$

$\frac{14}{5}x = \frac{70}{5}$

$x = 5$

$y = -\frac{4}{5}(5) = -4$

3 $f(5,-4) = 5^2 + (5)(-4) + (-4)^2$

$= \boxed{21}$

4. (2 points) The demand functions for a pair of commodities are given. Use partial derivatives to determine whether the commodities are substitute, complementary, or neither.

$D_1 = 2000 + \frac{100}{p_1+2} + 25p_2;\ D_2 = 1500 - \frac{p_2}{p_1+7}$ $= 1500 - p_2(p_1+7)^{-1}$

$\frac{\partial D_1}{\partial p_2} = 25 > 0$

$\frac{\partial D_2}{\partial p_1} = p_2(p_1+7)^{-2} = \frac{p_2}{(p_1+7)^2} > 0$

$\boxed{\text{Substitute commodities}}$

5. (5 points) Find the critical points of the given function and classify each as a relative maximum, a relative minimum, or a saddle point.

$f(x, y) = 4 + x^3 + y^3 - 3xy$

① $f_x = 3x^2 - 3y$

$f_{xx} = 6x$

$f_{xy} = -3$

$f_y = 3y^2 - 3x$

$f_{yy} = 6y$

$D(x,y) = f_{xx}f_{yy} - (f_{xy})^2$

$= (6x)(6y) - (-3)^2$

$= 36xy - 9$

② Critical points:

$f_x = 0$: $3x^2 - 3y = 0 \rightarrow x^2 - y = 0 \rightarrow y = x^2$

Plug in

$f_y = 0$: $3y^2 - 3x = 0 \rightarrow y^2 - x = 0$

$(x^2)^2 - x = 0$

$x^4 - x = 0$

$x(x^3 - 1) = 0$

$x(x-1)(x^2+x+1) = 0$

$x = 0 \quad x = 1$

$x = 0$: $y = (0)^2 = 0$

$x = 1$: $y = (1)^2 = 1$

$\underbrace{(0,0) \qquad (1,1)}_{\text{Critical pts.}}$

③ $(0,0)$:

$D(0,0) = 36(0)(0) - 9 = -9 < 0 \rightarrow$ $\boxed{(0,0) \text{ is a saddle point}}$

$(1,1)$:

$\left.\begin{array}{l} D(1,1) = 36(1)(1) - 9 = 27 > 0 \\ f_{xx}(1,1) = 6(1) = 6 > 0 \end{array}\right\} \rightarrow$ $\boxed{(1,1) \text{ is a relative min}}$