## Linear functions

The **slope** of a line is a number that measures the line's **steepness**.

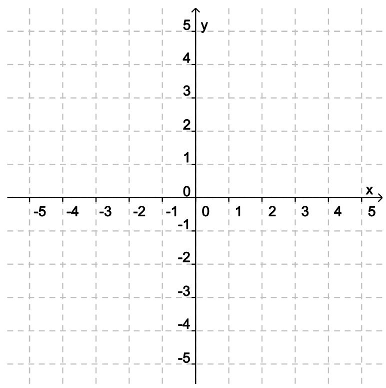
**Slope:** $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}$ ← **$(x_1, y_1)$ and $(x_2, y_2)$ are points on the line**

**Slope-intercept form:** $y = mx + b$ ← **$m$ is slope, and $b$ is $y$-intercept**

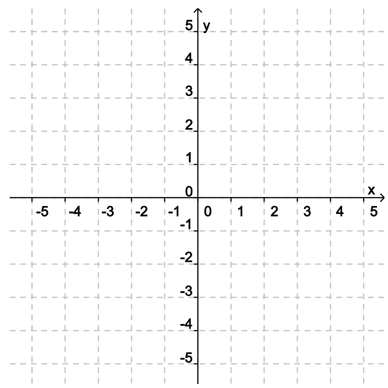
**Point-slope form:** $y - y_1 = m(x - x_1)$ ← **$(x_1, y_1)$ is point on line, and $m$ is slope**

**Ex 1.**

Graph $y = \frac{1}{2}x - 3$

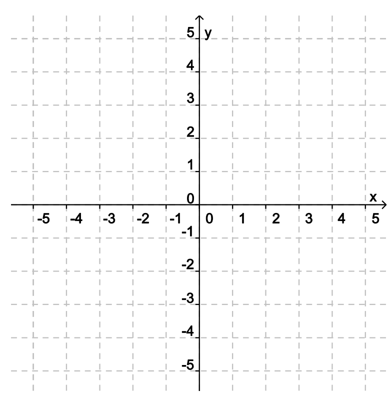


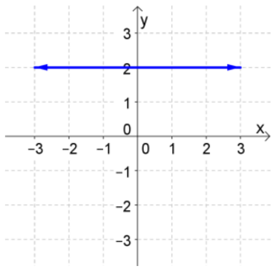
Graph $f(x) = -\frac{2}{3}x + 2$



**Ex 2.**

Find an equation of the line through $(2, 0)$ and $(-1, -6)$. Then rewrite the equation in slope-intercept form. Finally, graph the line.

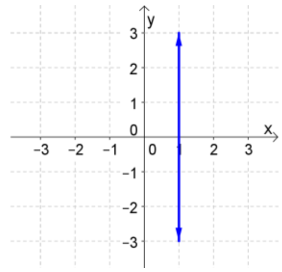





Horizontal line

Equation: ____________

Slope: _________




Vertical line

Equation: ____________

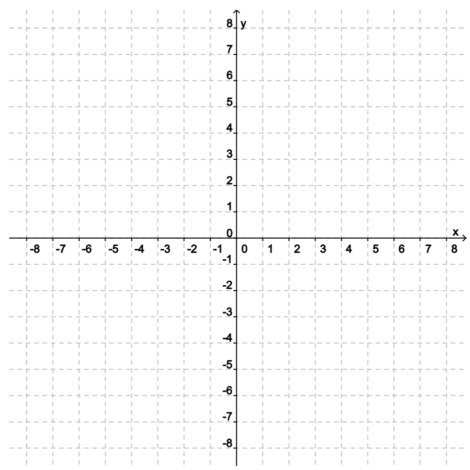
Slope: _____________

## Quadratic functions

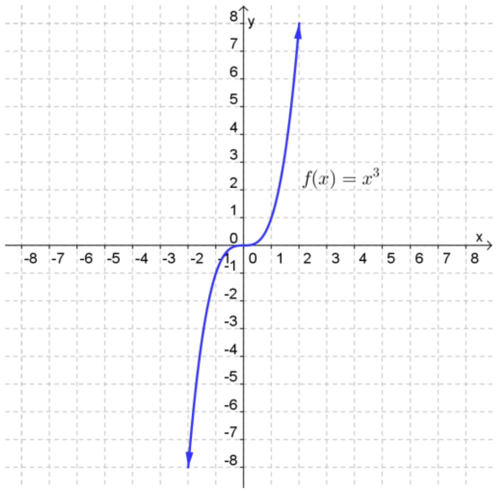
The graph of a quadratic function $f(x) = ax^2 + bx + c$ is a _________________.

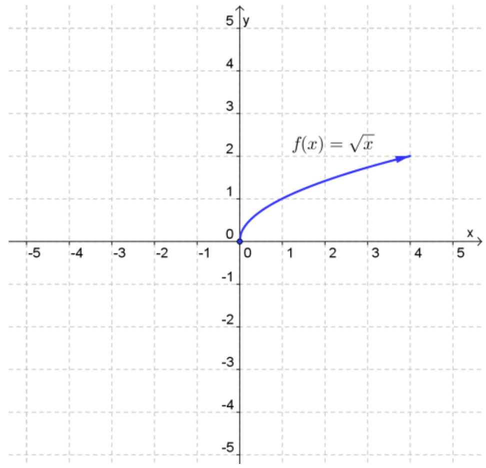
**Ex 3.**

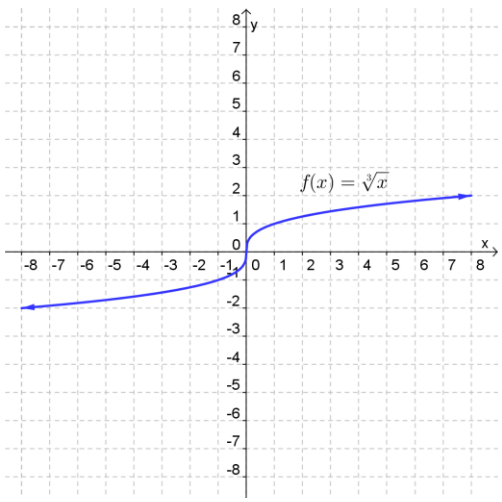
Graph $f(x) = -x^2 + 4x + 1$.

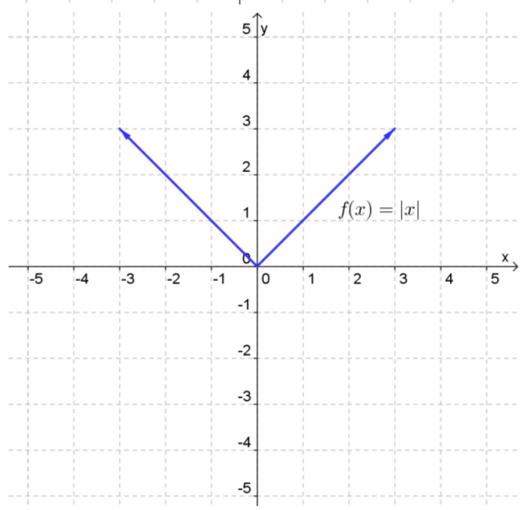


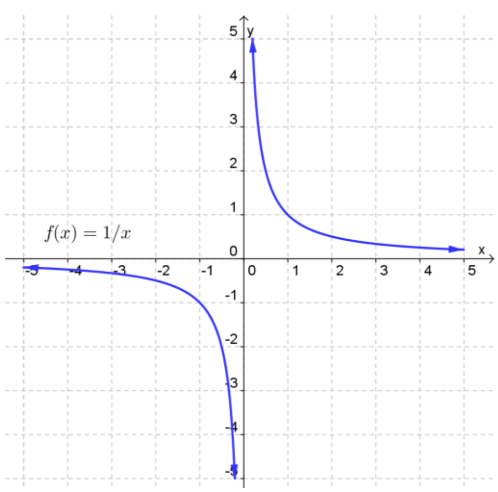
## Other functions

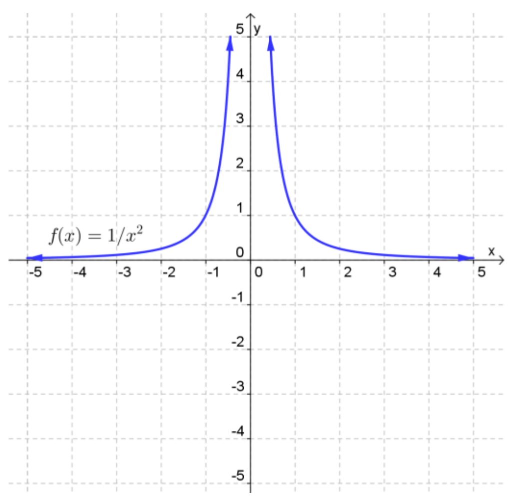


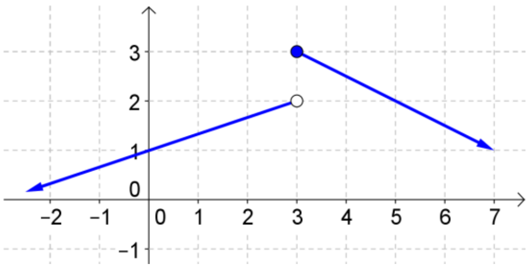
## Piecewise-defined function

**Ex 4.**

Suppose $f(x) = \begin{cases} \frac{1}{x-2} & \text{if } x < -2 \\ \sqrt{x^2+7} & \text{if } -2 \le x \le 1 \\ x^2 + 2x + 3 & \text{if } x > 1 \end{cases}$

$f(2) =$ $f(-4) =$

$f(1) =$ $f(-2) =$




To represent graph to the left, we could define

$$f(x) = \begin{cases} \frac{1}{3}x + 1 & x < 3 \\ -\frac{1}{2}x + \frac{9}{2} & x \ge 3 \end{cases}$$

We'll use piecewise-defined functions as examples when talking about limits (sections 1.5/1.6).

## Function composition

**Ex 5.**

Find $f(x-2)$ where $f(x) = (x+2)^4 - 3x^2$.

## Domain

The domain of a function is the set of all possible numbers you can plug into the function.

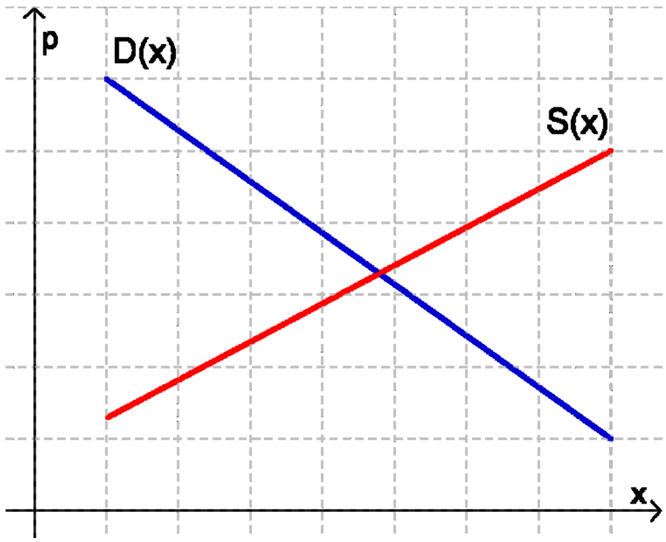
$\frac{}{\square} \rightarrow \square \ne 0$ $\sqrt{\square} \rightarrow \square \ge 0$ $\log \square \rightarrow \square > 0$

**Ex 6.**

Find the domain of $f(x) = \frac{2x^2}{x^2-1}$. Find the domain of $g(x) = \sqrt{3x+2}$.

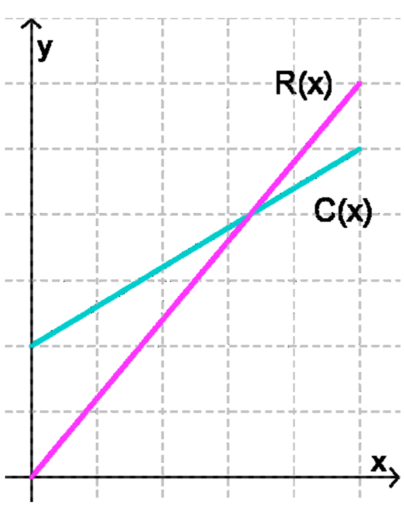
Here are some common business-related functions we'll be using throughout the course:
(note that $x$ represents a certain number of units or items, like $x = 10$ socks, or $x = 3$ bananas)

$\boldsymbol{D(x)}$ – **Demand** function is unit price that must be charged so $x$ units are sold
$\boldsymbol{S(x)}$ – **Supply** function is unit price at which producers are willing to supply $x$ units




The point where the demand and supply curves meet is called the _______________________.
Note: Sometimes $p(x)$ is used instead of $D(x)$ to represent the unit price.

$\boldsymbol{R(x)}$ **– Revenue** obtained from selling $x$ units
Note: $\boldsymbol{R(x) = x \cdot p(x)}$ (that is, # of units times price per unit)
$\boldsymbol{C(x)}$ – **Cost** of producing $x$ units
$\boldsymbol{P(x)}$ **– Profit** obtained from selling $x$ units
Note: $\boldsymbol{P(x) = R(x) - C(x)}$




The point where the revenue and cost curves meet is called the _________________________.

## Factoring

You'll want to brush up on factoring. Here is a summary of how to factor just about anything.

**How to Factor**

Always first factor out **GCF** (with negative if appropriate).

ex: $6x^4 + 3x^2 - 9x = 3x(2x^3 + x - 3)$

If **two** terms, try one of these:

$A^2 - B^2 = (A + B)(A - B)$ ex: $x^2 - 9 = (x + 3)(x - 3)$

$A^3 + B^3 = (A + B)(A^2 - AB + B^2)$ ex: $x^3 + 8 = (x + 2)(x^2 - 2x + 4)$

$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$ ex: $x^3 - 27 = (x - 3)(x^2 + 6x + 9)$

If **three** terms:

1. Check if you can use one of these:

   $A^2 + 2AB + B^2 = (A + B)^2$ ex: $x^2 + 6x + 9 = (x + 3)^2$

   $A^2 - 2AB + B^2 = (A - B)^2$ ex: $x^2 - 14x + 49 = (x - 7)^2$

2. Factor by thinking about product and sum

   ex: $x^2 - 5x + 6$ (Think: two #'s whose product is +6 and whose sum is -5. They're -2 and -3.)

   So, $x^2 - 5x + 6 = (x - 2)(x - 3)$.

   OR

   Factor by $\boldsymbol{ac}$-**method** (look for factors of $ac$ that sum to $b$).

   ex: $2x^2 + 3x - 5$ (Think: (2)(-5)=-10. Two #'s that multiply to -10 and add to +3 are -2, +5.)

   So, $2x^2 + 3x - 5 = 2x^2 - 2x + 5x - 5$

   $= 2x(x - 1) + 5(x - 1)$

   $= (x - 1)(2x + 5)$

If **four** terms, factor by **grouping**.

ex: $x^3 + 2x^2 - 4x - 8 = x^2(x + 2) - 4(x + 2) = (x + 2)(x^2 - 4)$

After factoring, check if any factors can be factored further.

ex: In the previous example, we should factor the $x^2 - 4$ so that:

$x^3 + 2x^2 - 4x - 8 = (x + 2)(x + 2)(x - 2)$

## Radicals and Rational Exponents

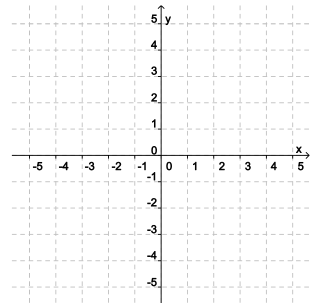
You'll also need to know how to rewrite expressions using rational exponents (recall that rational numbers are fractions with integers on top and bottom, for example $\frac{2}{3}, -\frac{7}{10}, 13, 0$).

$\sqrt{x} =$ _____ $\sqrt[3]{x} =$ ______ $\sqrt[4]{x} =$ ______

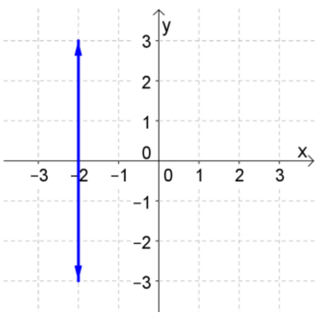
$\sqrt[3]{x^2} =$ ______ $\frac{1}{x} =$ ______ $\frac{1}{x^2} =$ ______

## Practice

1. Find an equation of the line through $(-3, 5)$ with slope $-2$. Then rewrite the equation in slope-intercept form. Finally, graph the line.

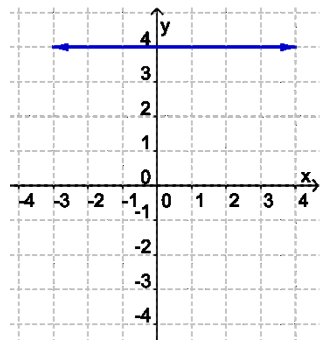



2. Write the equations and slopes for the following lines.




Equation: ___________

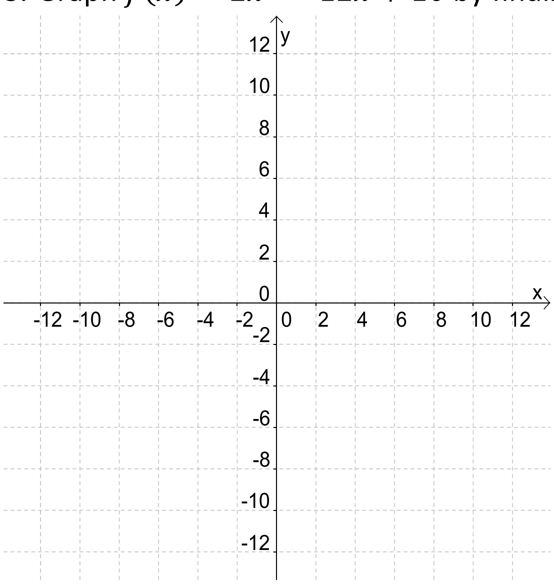
Slope: ___________




Equation: ___________

Slope: ___________

3. Graph $f(x) = 2x^2 - 12x + 10$ by finding and plotting the vertex, $x$-intercepts, and $y$-intercept.




4. Suppose $f(x) = \begin{cases} -5 & \text{if } x < -1 \\ x^2 + 1 & \text{if } -1 \le x < 2 \\ 2 - x & \text{if } x \ge 2 \end{cases}$. Find $f(1)$, $f(2)$, and $f(-2)$.

5. Find $\frac{f(x+h)-f(x)}{h}$ where $f(x) = 5x + 2$. Simplify.

6. Suppose the demand function for $x$ thousand units of a keyboard is:

$p(x) = -0.27x + 51$ (in dollars)

and the cost of producing $x$ thousand units is:

$C(x) = 2.23x^2 + 3.5x + 85$ (in thousands of dollars).

Find the revenue and profit functions.

7. Factor completely.

a) $x^2 - 4x + 3$

b) $x^2 + x - 6$

c) $x^2 - 4x$

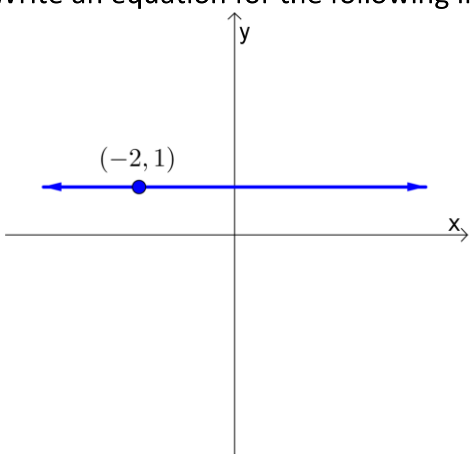
d) $8x^3 - 8x$

e) $6x^2 + 6x - 12$

f) $12x^3 - 3x$

## Homework

1. Find an equation of the line through $(0, -4)$ with slope $\frac{2}{3}$. Then rewrite the equation in slope-intercept form. Finally, graph the line.
2. Find an equation of the line through $(2, 5)$ with slope $-1$. Then rewrite the equation in slope-intercept form. Finally, graph the line.
3. Find an equation of the line through $\left(-1, \frac{1}{2}\right)$ and $(0, -\frac{2}{3})$. Then rewrite the equation in slope-intercept form.
4. Graph the line $x = -2$. What is the slope of the line?
5. Graph the line $y = 6$. What is the slope of the line?
6. Write an equation for the following line. What is the slope of the line?




7. Graph $f(x) = x^2 - 2$ by finding and plotting the vertex, $x$-intercepts, and $y$-intercept.
8. Graph $f(x) = 2x^2 + 4x - 6$ by finding and plotting the vertex, $x$-intercepts, and $y$-intercept.
9. Suppose $f(x) = \begin{cases} 1 - 3x & \text{if } x \le 4 \\ \frac{2}{x} & \text{if } x > 4 \end{cases}$. Find $f(8)$, $f(0)$, and $f(-1)$.
10. Suppose $f(x) = \begin{cases} \sqrt{x + 14} & \text{if } x < -6 \\ 2x^2 + 1 & \text{if } -6 \le x \le -1 \\ \sqrt[3]{x} & \text{if } x > -1 \end{cases}$. Find $f(-10)$, $f(-6)$, $f(-1)$, $f(0)$, and $f(27)$.
11. Find $f(x + 1)$ where $f(x) = x^2 - 3x + 2$. Simplify.
12. Find $f(x + h)$ where $f(x) = \frac{x}{x+1}$.
13. Find $\frac{f(x+h)-f(x)}{h}$ where $f(x) = 2x - 3$. Simplify.
14. Find $\frac{f(x+h)-f(x)}{h}$ where $f(x) = x^2$. Simplify.
15. Find the domain of $f(x) = \frac{x+1}{x^2-x-2}$.
16. Find the domain of $f(x) = \sqrt{6 - 2x}$.
17. Find the domain of $f(x) = \frac{x-1}{\sqrt{x+2}}$.
18. Factor completely.
    a) $3x^4 + 4x^3$ b) $12x^2 + 36x - 48$ c) $6x^2 + 6x - 12$ d) $x^2 - 2x + 1$ e) $4x^2 - 9$
    f) $x^3 - 4x^2 - 3x + 12$
19. Rewrite the following using rational exponents.
    a) $\sqrt{x}$ b) $\sqrt[4]{x}$ c) $\sqrt[5]{x^2}$ d) $\frac{3}{x}$ e) $\frac{5}{x^2}$ f) $\frac{1}{x^5}$ g) $\frac{1}{\sqrt{x}}$
20. Rewrite the following using radicals or fractions.
    a) $x^{\frac{1}{3}}$ b) $x^{\frac{3}{4}}$ c) $x^{\frac{1}{2}}$ d) $4x^{-1}$ e) $x^{-3}$ f) $x^{-\frac{2}{3}}$

21. Suppose the demand function for $x$ hundred units of a textbook is:
    $D(x) = -0.37x + 47$ (in dollars)
    and the cost of producing $x$ hundred textbooks is:
    $C(x) = 1.38x^2 + 15.15x + 115.5$ (in hundreds of dollars).
    Find the revenue $R(x)$ and profit $P(x)$.
22. Suppose that when the price of a water bottle is $p$ dollars per unit, then $x$ thousand units will be purchased by consumers, where $p = -0.01x + 4$. The cost of producing $x$ thousand bottles is $C(x) = 0.005x^2 + x + 120$ thousand dollars.
    a) Find the profit function, $P(x)$. (Notice that big $P$ and little $p$ mean two different things. Like passwords, math is case sensitive! ☺)
    b) Using $P(x)$, determine the level of production $x$ that results in maximum profit.
    c) What unit price $p$ corresponds to maximum profit?
23. You start a company that produces mechanical pencils whose lead does not break while writing. Your total cost consists of a fixed overhead of $6000 plus production costs of $5 per pencil.
    a) Find the total cost function $C(x)$, where $x$ is the number of pencils produced.
    b) Sketch the graph of $C(x)$.
24. You're in the cut-throat iPhone case business. You estimate that you can sell a case for $2 more than it costs to produce it. Your fixed costs are $11500.
    a) Express total profit $P(x)$ as a function of the level of production $x$ (that is, $x$ represents the number of cases produced).
    b) How much profit (or loss) is generated when 5000 cases are produced?
    c) What is the level of production at the break-even point? (Hint: Recall that the break-even point occurs when $R(x) = C(x)$. Equivalently, it happens when $P(x) = 0$.)
25. Suppose the supply and demand functions are $S(x) = 4x + 200$ and $D(x) = -3x + 480$, respectively. Find the value of $x_e$ for which equilibrium occurs and the corresponding equilibrium price $p_e$.

Note: Please visit davidsmath.com/math140 for an answer key to the homework.